Experimental Study of the Dynamics of Two Coupled Modified Wienbridge Oscillators

A Project report submitted for the academic year 2012-2013

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IISER KOLKATA

By

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CERTIFICATE

This is to certify that this project entitled 'Experimental Study of the Dynamics of Two Coupled Modified Wienbridge Oscillator' submitted as a project report for the academic year 2012-13 of the BS-MS Dual Degree programme at Indian Institute of Science Education and Research, Kolkata represents original research carried out by Abhijit Chakraborty during the month of June-July 2013, at NIT Sikkim under the supervision of Dr. Md. Nurujjaman, Physics Department, NIT Sikkim.

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Abhijit Chakraborty	Dr. Md. Nurujjaman

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Abstract

Synchronization is often important in non-linear models, be it biological systems or electronic circuits. The purpose of this paper is to show the synchronization between two modified Wienbridge oscillators coupled by a variable resistance. The coupling is done at fixed point and oscillatory phase, both of which show synchronization throughout the whole range of the coupling parameter. In the first case, both the oscillators jump to oscillatory phase. Also, though the detuning between the two circuits is high, complete synchronization and high order frequency locking has been observed in the second case. Besides, two coupled Wienbridge oscillators show interesting Lissajous figures in both the cases.

Contents

1. Introduction

The study of synchronization is an important area of non-linear dynamics .Now synchronization in chaotic oscillators [1] [2] is also an active field of discussion because it has applications in many areas of science and technology[3] [4]. In case of synchronization the systems are interrelated, be it frequency locking or phase entrainment. For interaction strength above a critical value the coupled oscillators show common frequency. The application of this phenomenon can be seen in cardiac cycles [5], neural oscillators [6] as well as in parasitology [7], behavioural psychology [8] or ecology [9]. The main theme of this project is also to investigate various aspects of synchronization. Coupling of two modified Wien-bridge oscillator produces synchronization and various order of frequency locking. This provides insight on the interesting field of non-linear dynamics and synchronization.

Wienbridge oscillator [10] is an important circuit used vastly in audio devices to minimize the effect of noise and to stabilize the system. But this circuit has also many important characteristics which makes Wienbridge oscillator an academic interest, such as, the effects of coupling this oscillator with other non-linear circuits. Many things are already known about Wienbridge oscillator. So the circuit has been modified a little bit and then the autonomous dynamics as well the dynamics when coupled with another identical oscillator with different natural frequency is studied.

The coupling is done at both the fixed point phase and oscillatory phase. In the first case coupling gives rise to oscillation in each circuit and the system remains in synchronization throughout. In the second case two highly detuned oscillators when coupled show a common frequency and the locking order increase with the increase of coupling parameter value. Both the cases give rise to various interesting Lissajous figures [11].

2. Study of Autonomous Dynamics

Wien-bridge oscillator is a feedback oscillator which consists of an op-amp and RC bridge circuit where the oscillating frequency is set by the R and C components [12]. But instead of the original Wienbridge oscillator we have used a modified one. The extra component, consisting of two diodes in anti-parallel and an 11 kOhm resistor parallel with them provides the circuit stability and also a source of non-linearity. Again the two resistors normally used to balance the bridge is replaced by a variable resistor of 20 kOhm.

We have used two circuits with different natural frequency. The natural frequencies are changed by varying the capacitors C_3 and C_4. The capacitors used in the two circuits are respectively 96 nF, 103 nF and 10nF, 9.56 nF.

The circuit diagram of our circuit is shown in Fig.1. The output is taken from the 6th terminal of the op-amp and is feed to the oscilloscope to see the output waveform and autonomous dynamics study.

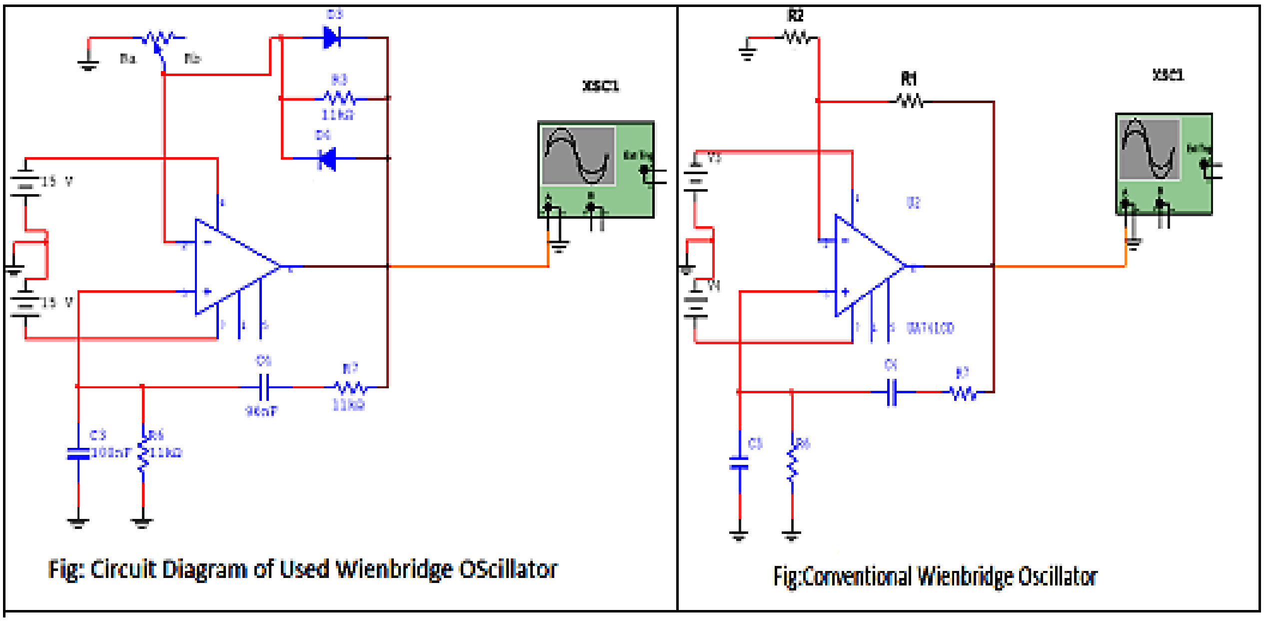

Fig: Circuit Diagram of Used Wienbridge OScillator

Fig:Conventional Wienbridge Oscillator

Fig: 1

Using the circuit shown in Fig. 1, we have studied the autonomous dynamics in both experiment and spice simulation. The time series plot (voltage vs. time plot) of the output gives rise to square waveform at the beginning. As we increase the resistance of the variable resistor the square waveform changes to sine waveform through some intermediate steps. Also the frequency varies as the resistance is increased. The variation of the frequency with resistance is shown in Fig.2.

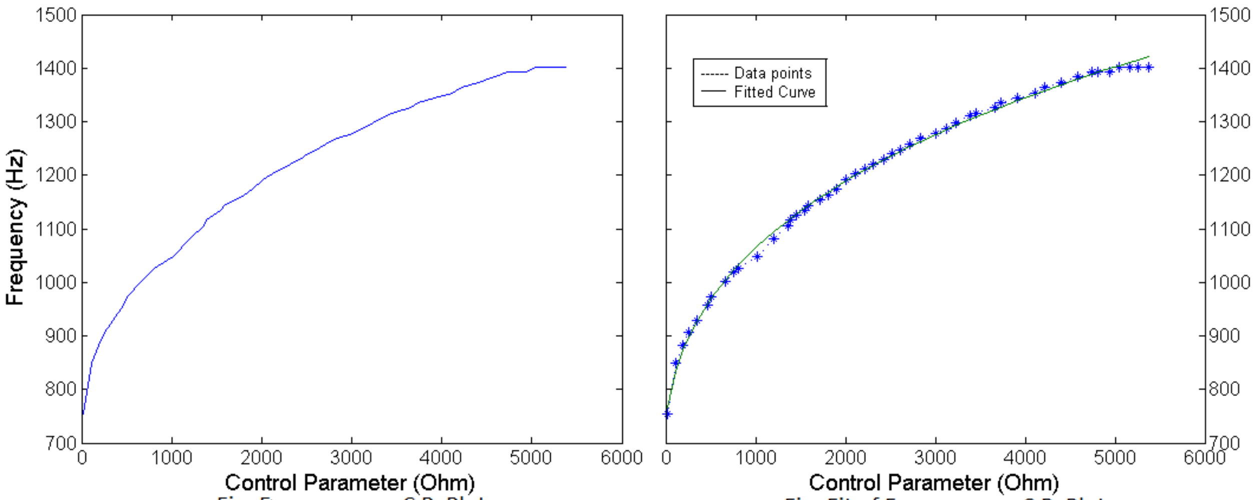

Fig: Frequency vs. C.P. Plot

Fig: Fit of Frequency vs C.P. Plot

Fig: 2

The frequency varies logarithmically with the variable resistor which we have considered as the control parameter. The function which is fit with the data points is of the form $a*(\log(x))^n + b$ with the following values and corresponding error.

Parameter	Value	Error
a	0.343829	+/- 0.05341 (15.53%)
b	753.852	+/- 6.475 (0.859%)
n	3.52059	+/- 0.06881 (1.955%)

Again the data points of the amplitudes vs. C.P. plot lead us to a bifurcation plot which is similar to pitch-fork bifurcation. This is expected as the increase of the C.P. gradually leads the system to a fixed point. The fixed point is a stable one. The amplitude denotes the peak values of the oscillations in voltage. The figure obtained from both experiment and simulation is given in Fig.3.

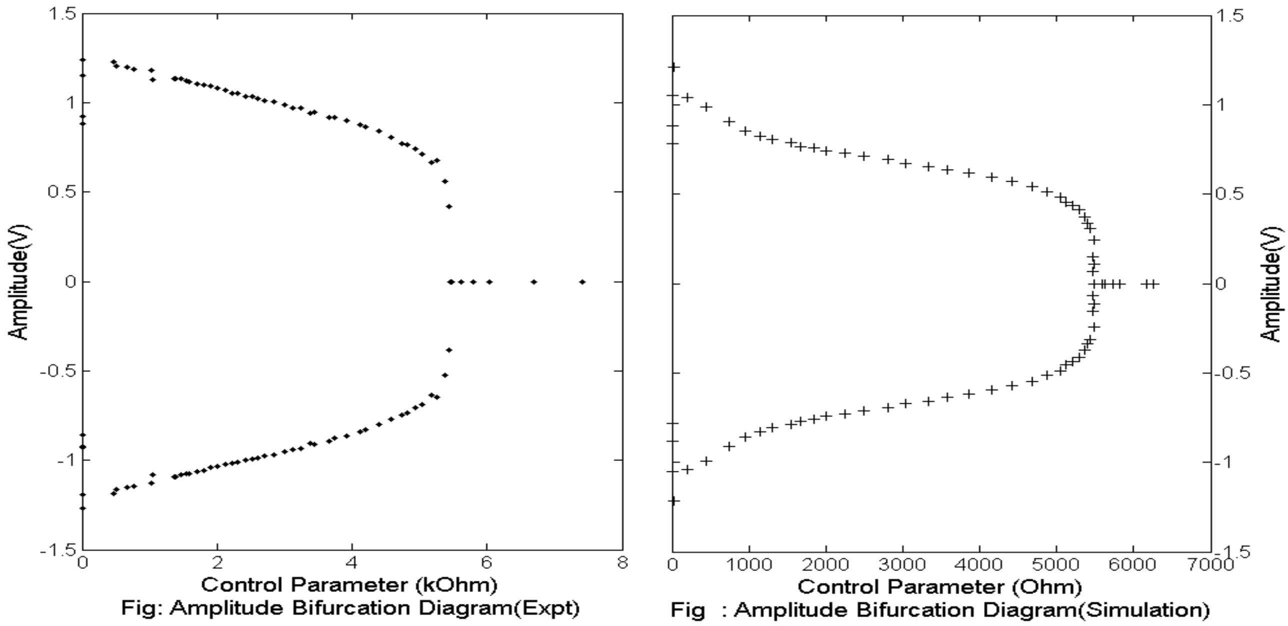

Fig: 3

The similarity between the experimental and simulation results shows the authenticity of the experiment. Now the results of mutual coupling of two modified Wienbridge oscillator can be studied, based on this platform.

3. Coupling of Two Oscillators

The autonomous dynamics of the modified Wienbridge oscillator is already described in the above section. Now various interesting phenomena occur when two oscillators are coupled by a variable resistor of 20 kOhm which is hereafter referred as the coupling parameter or C.P. There can be two cases when the coupling is done, one is when both the oscillators are at stable fixed point and another when both the oscillators are at oscillation with a large detuning between the frequencies of the two oscillators. The experimental data and the corresponding analysis of the phenomena occurring in both experimental set up and spice simulation is described in the following two sections. The circuit used to couple two oscillators is given in Fig.4.

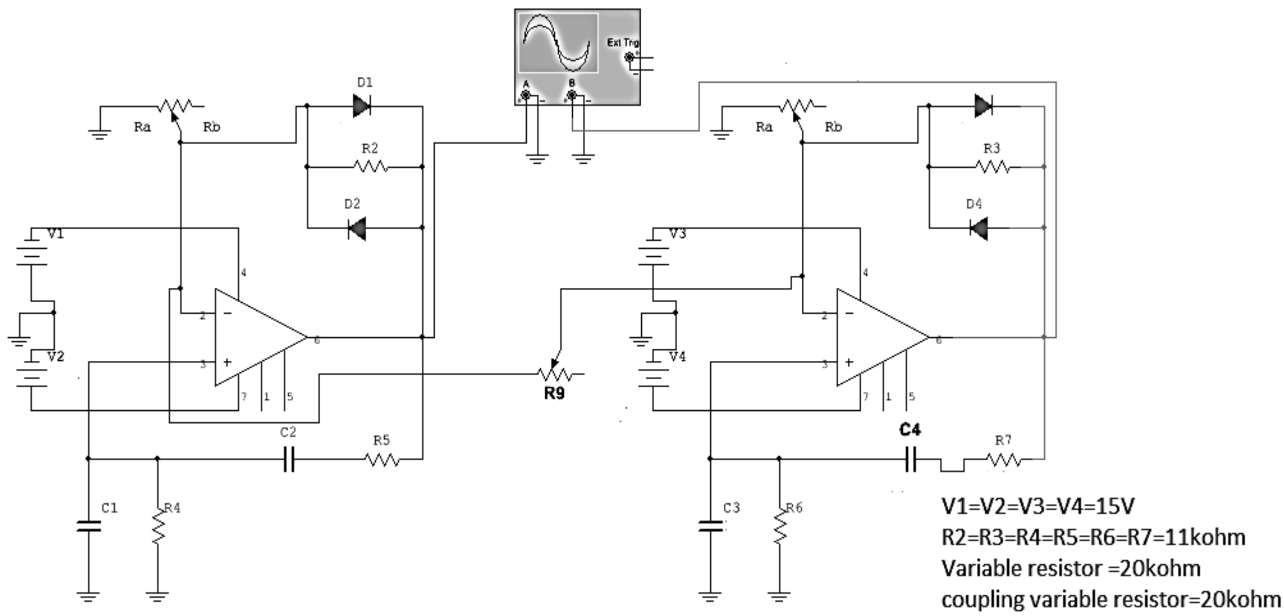

Fig: Circuit Diagram for Two Coupled Wienbridge oscillator

Fig: 4

To produce the detuning in the two circuits we have used different capacitors with C_1=C_2=10 nF and C_3=96.5 nF, C_4=103 nF. Producing the detuning is important because otherwise we cannot realise the effect of synchronization. The output of the circuit containing capacitors C_1, and C_2 (the circuit shown in the left of Fig.4) is hereafter referred as Channel A, and the other one with capacitors C_3, and C_4 is referred as Channel-B (the circuit shown in the right of Fig.4).

3.1. Both Oscillators at Fixed Point

The natural frequency of the two coupled circuits are different because of the capacitors in each circuit has a different value. Now both the oscillators are kept at fixed point, far from the oscillating phase. This is done so that a little amount of voltage will not displace the fixed point to an oscillatory phase. However as soon as they are coupled with a variable resistor both the oscillators jump to oscillation phase. Their waveforms are not exactly sine or square but something intermediate. The more interesting part is the frequency of the waveforms. Despite the fact that their natural frequencies differ by a large factor, the frequency of the waveforms are same for two oscillators, i.e., they are synchronized and mutually dependent on each other. Though the frequencies are same their phase differs from each other. The phase difference is nearly 180 degrees and the phase difference remains almost constant throughout the experiment. The time series plot of the outputs at highest coupling strength (lowest coupler resistance) are given in Fig.5 for the two circuits (the outputs of two circuits are denoted as Channel-A and Channel-B).

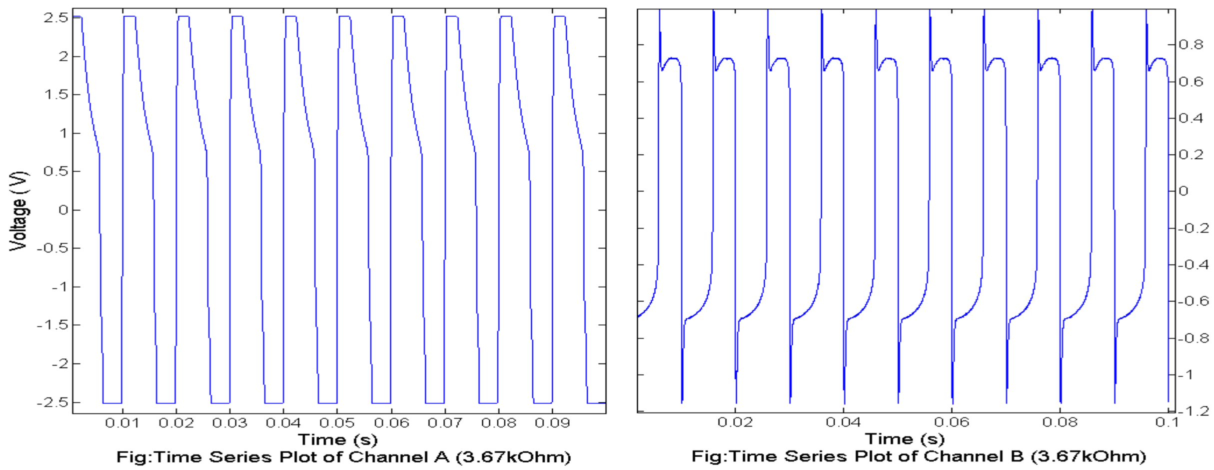

Fig: 5

Output in oscilloscope is taken from the 6th terminal of the op-amp of the two circuits. The mutual dependence of output can be seen from time series plots of channel A and channel B of the oscilloscope. At coupling parameter 4.94 kOhm the nearly square waveform of channel B starts to form sine wave gradually. Due to the influence on each other at the same resistance channel A shows kinks in the waveforms induced by the sine wave of channel B. The peak of the kinks coincide with the peak of the newly generated sine waves. The time series plots are shown in the following figure (Fig.6).

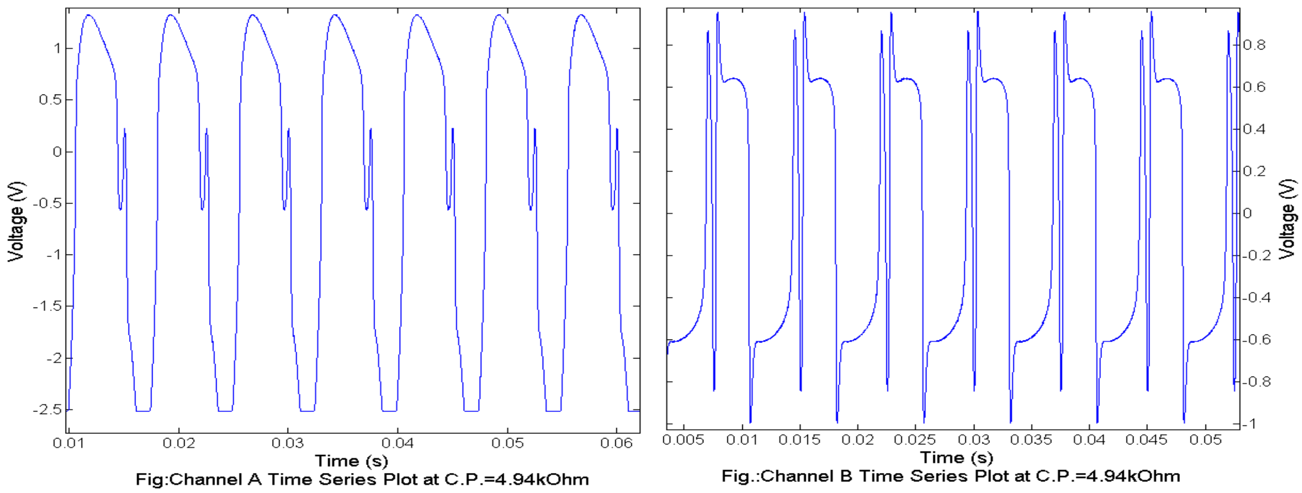

Fig: 6

As the coupling parameter is increased, i.e., the coupling strength is decreased, the waves similar to square waves gradually transform into sine waves. At resistance 6.78 kOhm sine waves with modulation arise. The frequency of the sine waves are greater than the frequency of the waves similar to square waves. So, from the plot of the coupling parameter vs. frequency, the sudden frequency jump at 6.78 kOhm can be seen. We have considered the dominant frequency for the C.P. vs. frequency plot. Though the square waves die out gradually, the FFT plot of the data points above C.P.>6.78 kOhm shows the frequency of the square wave. This is because the modulating frequency of the sine waves is equal to that of the square waves. The modulating frequency remains same for rest of the data taken but the frequency of the sine waves gradually increase though the increase is very slight. The value of the modulating frequency is F=156.4 Hz and the sine wave frequency ranges from 1198-1354 Hz. The frequency jump for both the channels are shown in Fig.7.

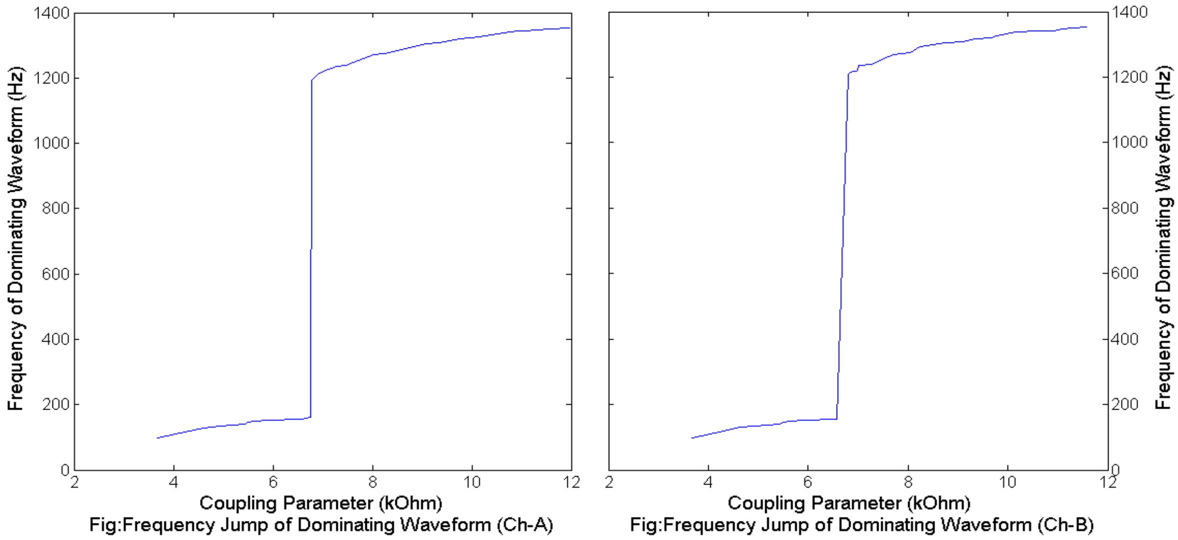

Fig: 7

The frequency of the waves similar to square waves are hereafter referred as modulating frequency and the other one is referred as sine wave harmonic. Fig.8 and Fig.9 clearly show the presence of two distinct frequencies at coupling resistance 6.78 kOhm. The modulating frequency is weaker at and after 6.78 kOhm and so the sine wave harmonic is considered the dominating waveform hereafter. This explains the sudden frequency jump at 6.78 kOhm in Fig.7.

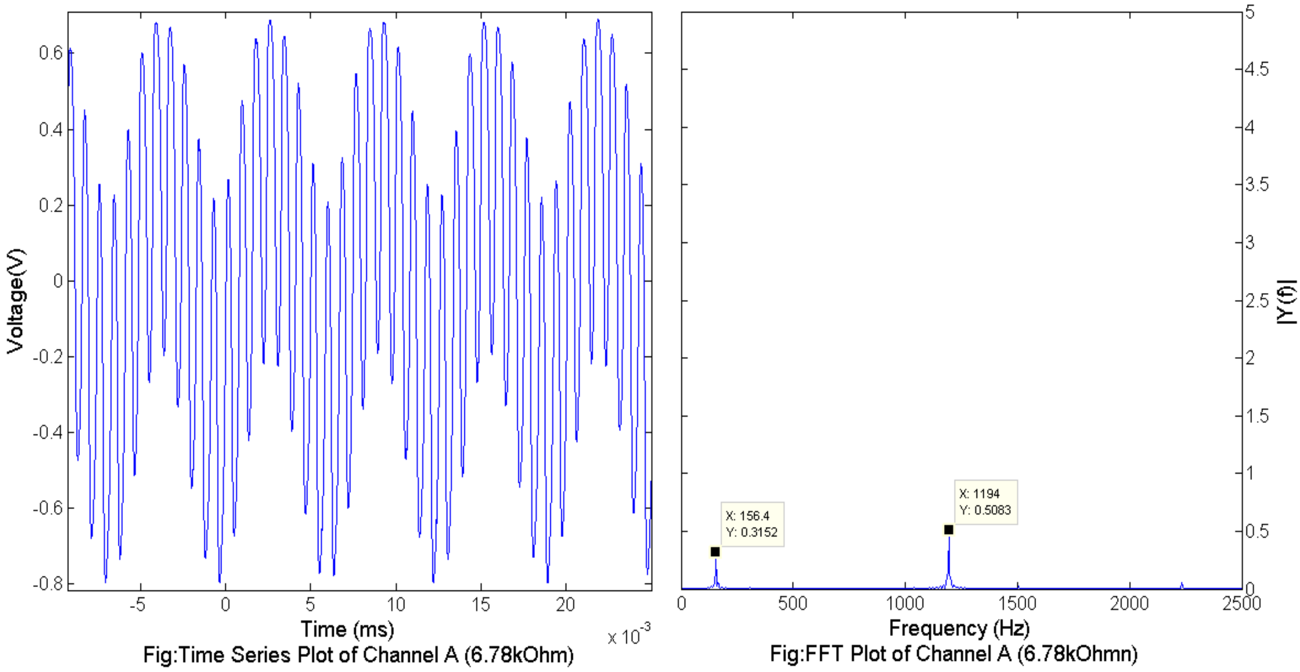

Fig: 8

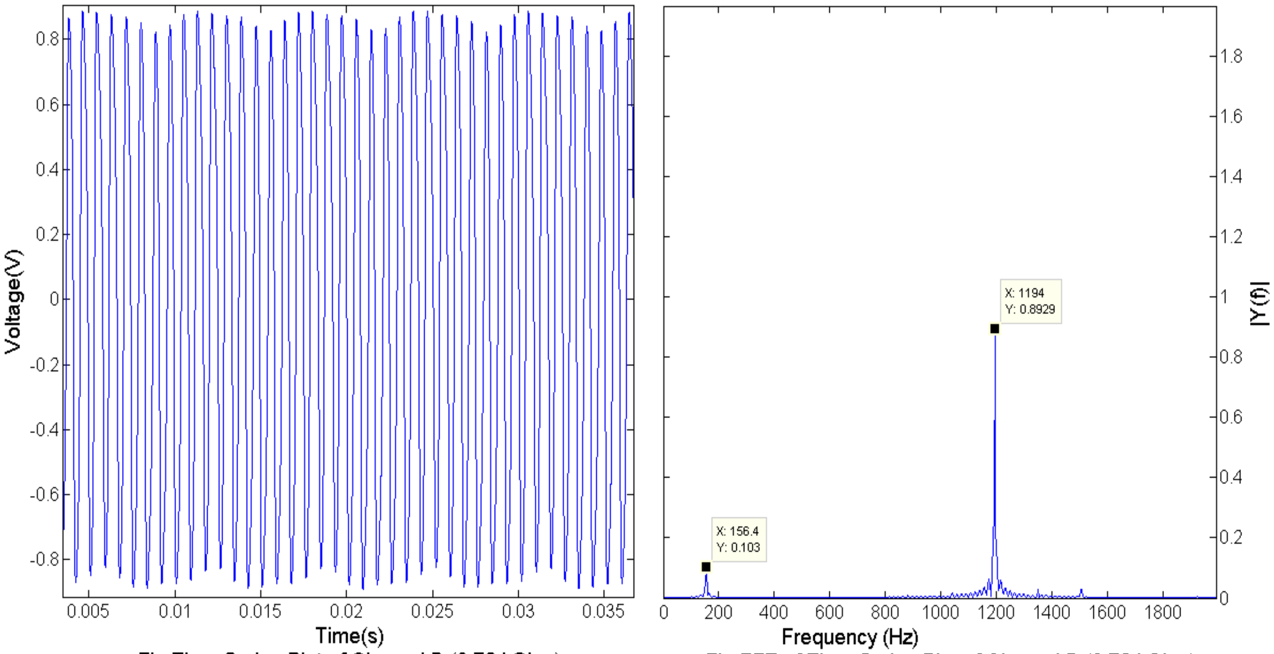

Fig:Time Series Plot of Channel B (6.78 kOhm)

Fig:FFT of Time Series Plot of Channel B (6.78 kOhm)

Fig: 9

Fig.8 and Fig.9 also show that the modulation in channel A and Channel B are not exactly same and they differ in amplitude. The modulation is more distinct in channel A rather than channel B. The corresponding amplitudes of the two frequencies (the modulating frequency and the sine wave harmonic) obtained from the FFT plots are different and can be plotted against the coupling parameter to show the difference.

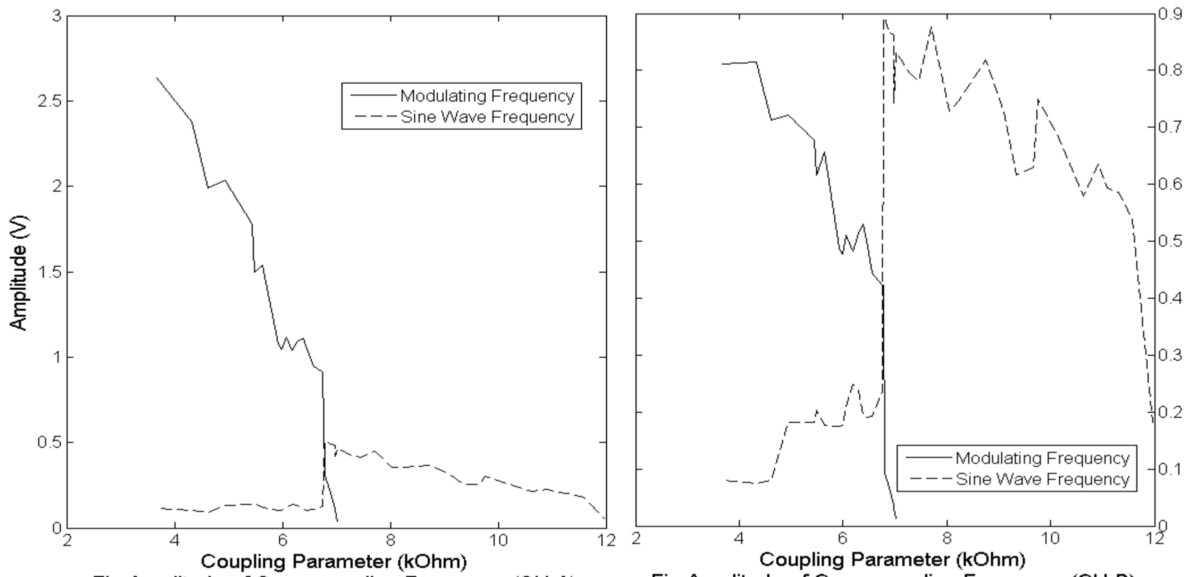

Fig:Amplitude of Corresponding Frequency (CH-A)

Fig:Amplitude of Corresponding Frequency (CH-B)

Fig: 10

Fig.10 also shows the decaying amplitude of the modulating frequency and the sudden jump in the amplitude of sine wave harmonic. The jump is at 6.78 kOhm, at which the modulation in sine waves sets into action. The modulating frequency also dies out first as the coupling resistance is increased. The sine wave harmonic also decays after a value of nearly 12 kOhm at which fixed point appears.

This is because as we increase the coupling parameter the coupling strength decreases and finally becomes very less and can't even effect each other. Hence finally fixed point approaches which was the initial state of both the oscillator circuits.

The four plots in Fig.11 show the variation of the modulating frequency and the sine wave harmonic for both channels with the change of the coupling resistance.

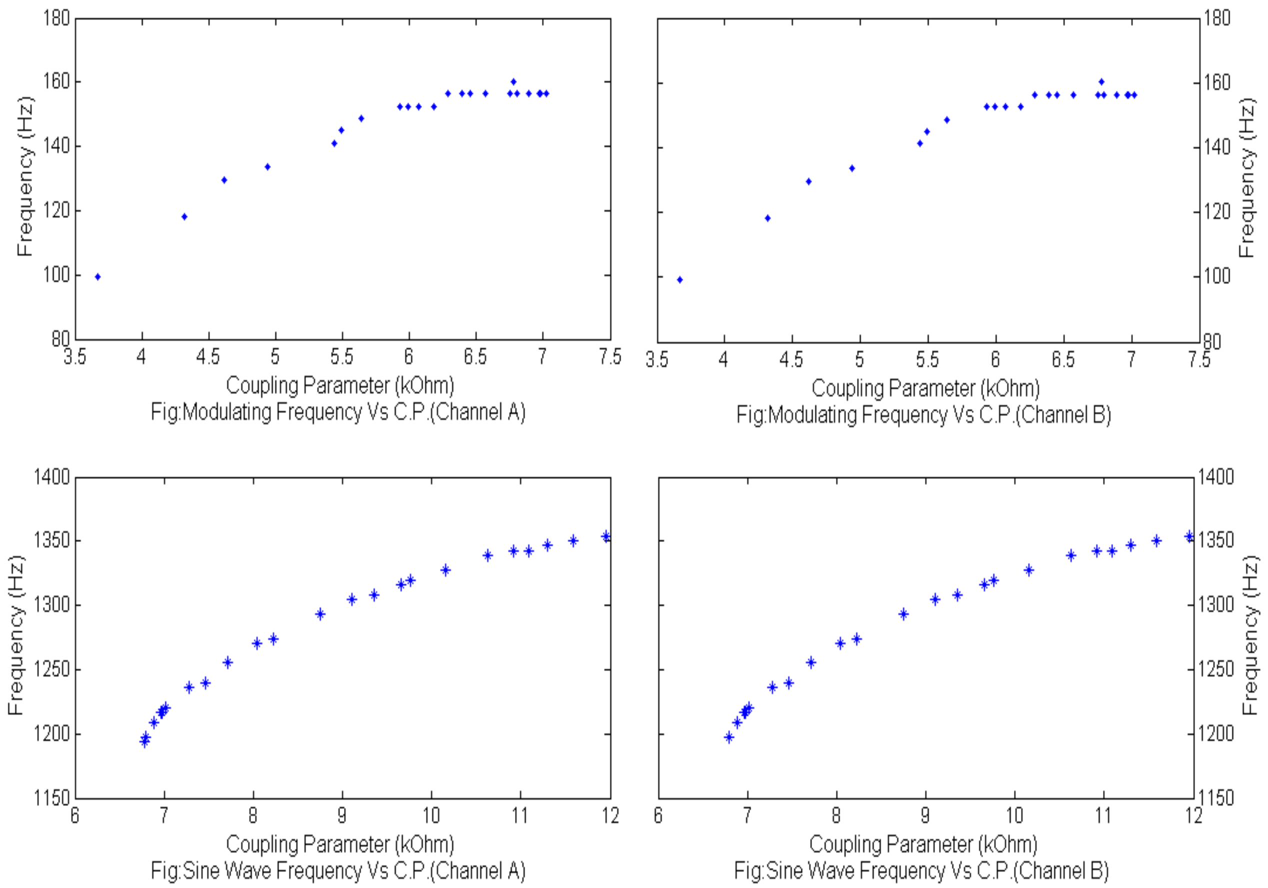

Fig: 11

As we can see the modulating frequency gets saturated as the C.P. is increased, whereas the sine wave harmonic experiences a steady increase and the plots corresponding to channel A and channel B are totally identical. So it clearly shows, despite of initial large detuning the two coupled systems are entrained throughout the experiment. The synchronization region is really large for this mutually coupled system.

Though there is entrainment between the two systems the amplitude vs. C.P. plots show that the initial amplitude of the modulating frequency is low for channel B. So the study of variation of delta amplitude vs. C.P. is important in this case. Fig.12 show the synchronization region and the continuous decay of the delta amplitude with the increase of C.P.

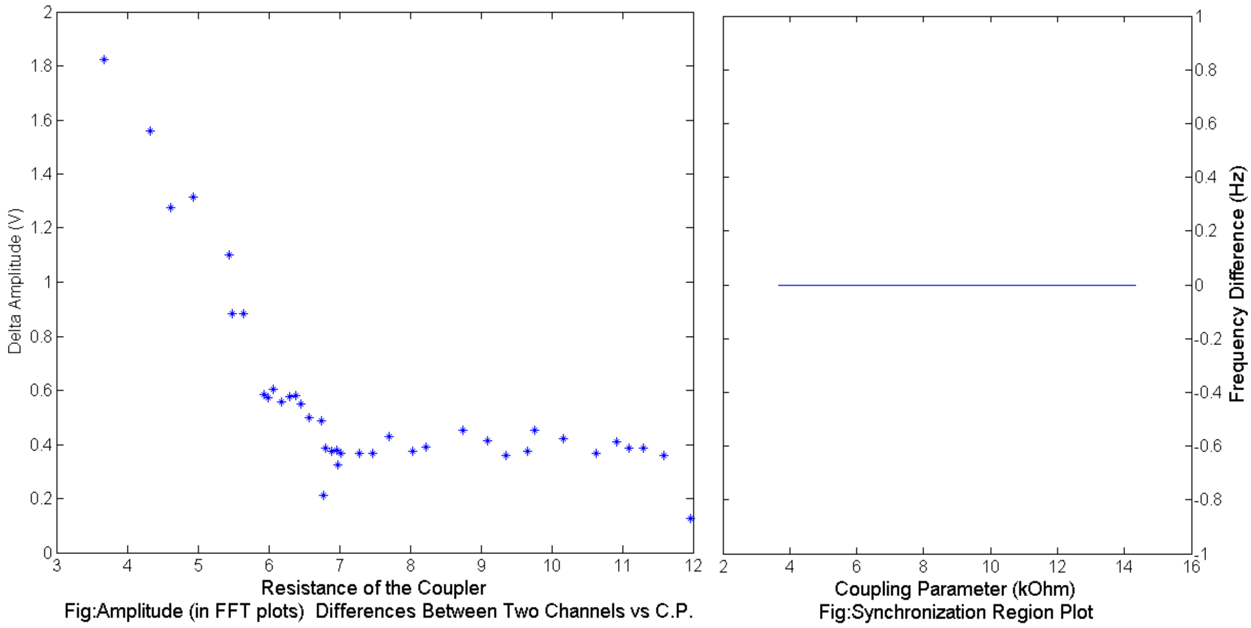

Fig: 12

The delta amplitude vs. C.P. plot shows that gradually the difference between amplitudes of the two channels approaches a constant value. This indicates amplitude locking in the mutually coupled system.

Though the whole region is synchronized it must be important to note that we have considered the modulating frequency here to plot the synchronization region. But it should be noted that after 6.78 kOhm the square-like waveform decays into sine waves with modulation. Now there is an important difference between synchronization and modulation. In the case of n:1 locking the effect of the coupling can be of two type:1) It can cause modulation and 2) It adjusts the average period of oscillation which leads to synchronization [13].

Now there can be synchronization with or without modulation. But in case of 1:1 locking modulation does not occur. So in case of limit cycle synchronization is without modulation. Here we can see both synchronization and modulation after C.P. = 6.78 kOhm. And this gives rise to various interesting Lissajous figures.

Another aspect of any non-linear electronic circuit is the phase space plot [14]. Here the plots also give rise to various interesting figures. Starting from a curve somewhat similar to a hysteresis curve the figure evolves to Lissajous figures leading towards limit cycle [15] and finally fixed point. All the figures are closed curves as it should be in case of quasi-periodic or periodic oscillations. Some of the interesting figures are shown in the following plots in Fig.13.

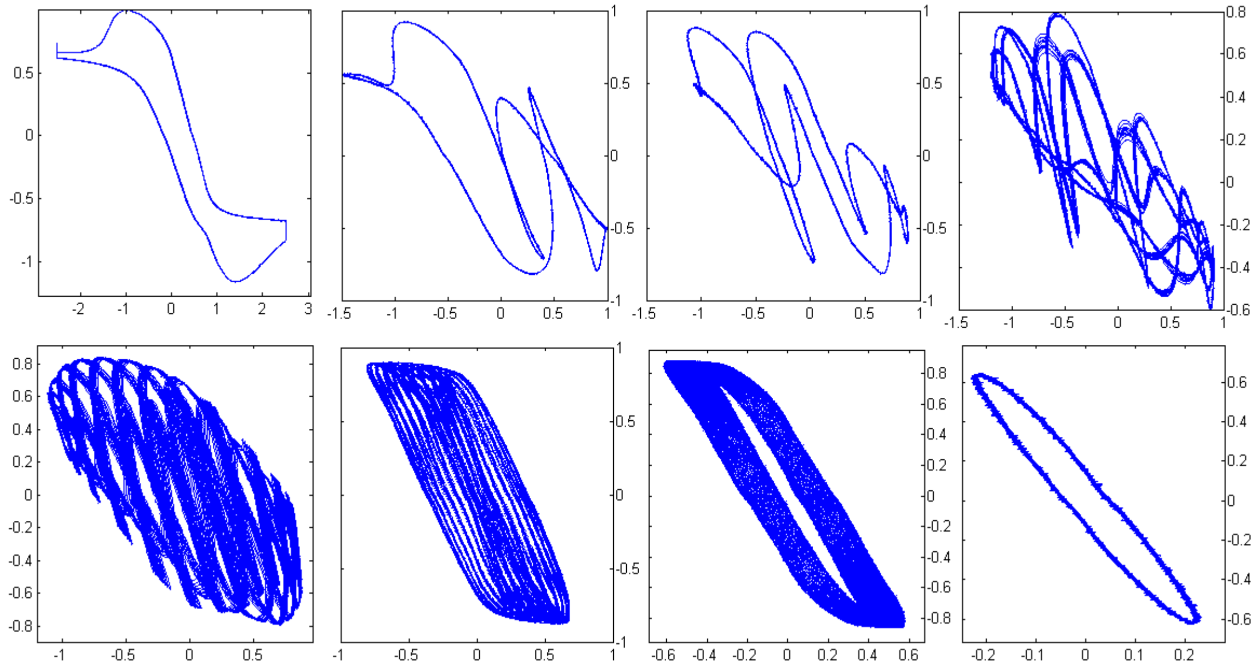

Fig:Lissajous Figures and Other Phase Space Plots

Fig: 13

The phase space plots are important to understand the loss of synchronization or rise of more than one frequency or quasi-periodic motion. In case of synchronous motion they give rise to closed curves known as Lissajous figures. Not only Lissajous figures help us in understanding the synchronization but also the relation between the frequencies. There can be many kind of synchronization. Such as the locking in the frequencies may be 1:1 or n:1 for any value of n. The 1:1 locking gives rise to the limit cycle is phase space plot while 2:1 locking indicates a plot looking similar to the no. 8. In case of quasi-periodic state the point never returns to same position and occupy the whole phase space.

Now all the figures shown in Fig.13 are closed curves, indicating synchronous motion which we have already shown. As the resistance is increased the complexity of the figures decrease gradually. This points to the fact as we increase the coupling parameter the relation between the two frequencies become less complex. The second and third figure suggests higher order locking which means high integer or fractal values of n. As the C.P. is further increased quasi-periodic motion can be observed. But at last the motion becomes periodic as well as the locking becomes 1:1. Finally when the coupling is very weak the oscillators arrive at fixed point as oscillation death occurs.

The following time series plots and their corresponding phase space plots in Fig.14 help us to understand the dynamics of the system in the transition phase of higher order locking to limit cycle (The dotted lines indicate Channel A while the bolder lines denote Channel B).

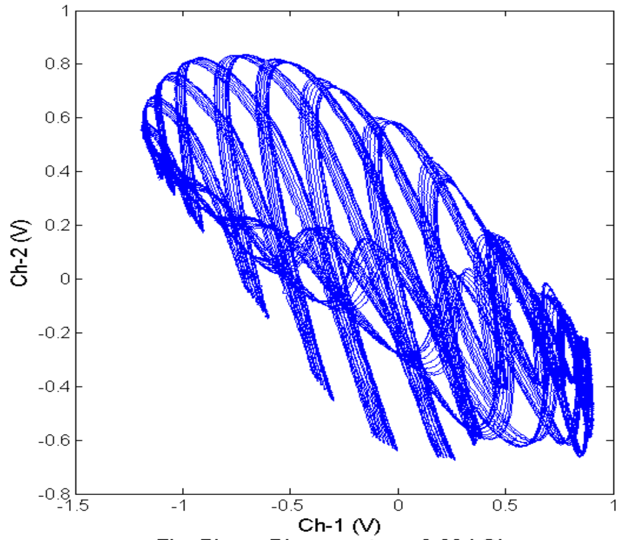

Fig: Phase Diagram at cp=6.63 kOhm

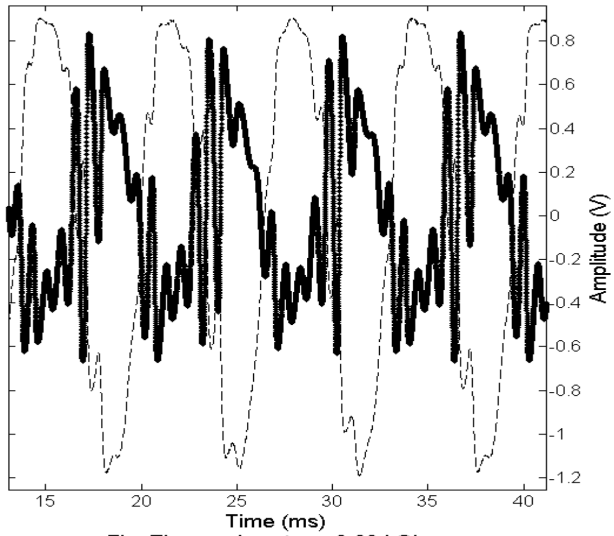

Fig: Time series at cp=6.63 kOhm

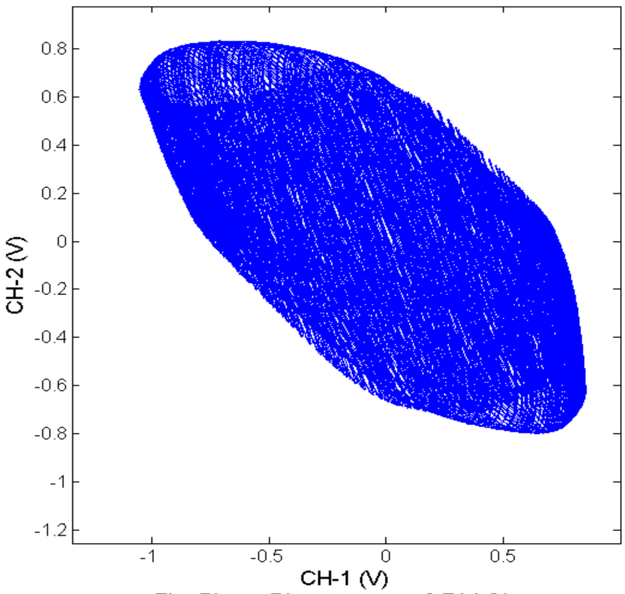

Fig: Phase Diagram at cp=6.71 kOhm

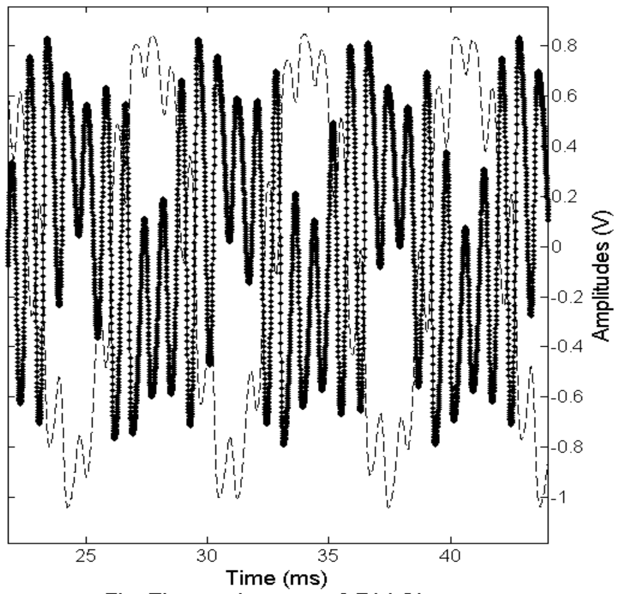

Fig: Time series at cp=6.71 kOhm

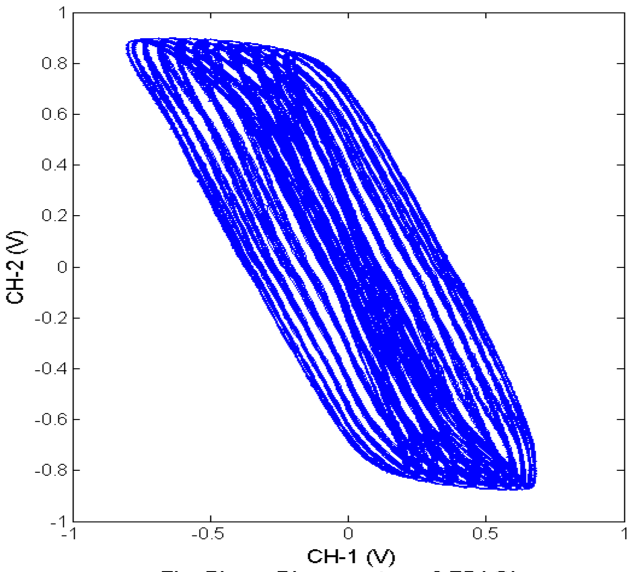

Fig: Phase Diagram at cp=6.75 kOhm

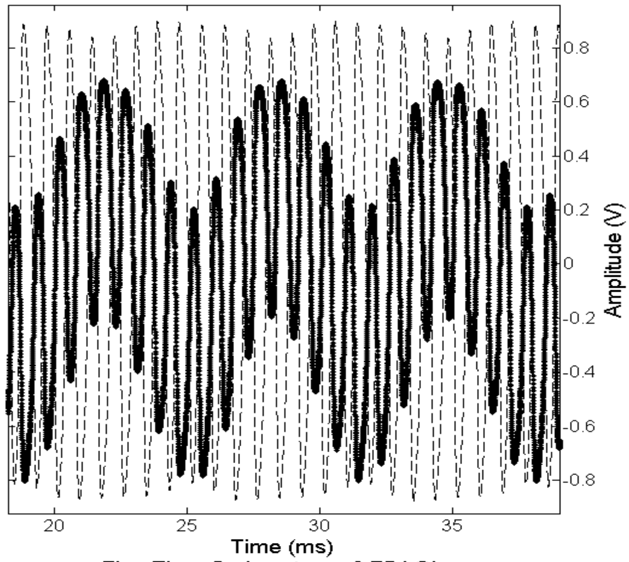

Fig: Time Series at cp= 6.75 kOhm

Fig: 14

The first plot implies complex order locking. As it can be seen, the oscillations are not totally periodic, rather it can be called quasi-periodic but it still is synchronized and we can see the effect of a channel on another. Each of the channel shows same type of behaviour. As the quasiperiodicity increases the phase space plot occupies the whole region which is depicted in the second figure. Now as the quasiperiodicity gradually evolves into the sine wave harmonics with oscillation the phase space plots tend to the toroidal Lissajous figure. This is nothing but many limit cycles aggregated together because of the existence of the modulating frequency. Now as stated earlier the modulating frequency dies out gradually and the toroidal figure transforms into limit cycle.

To summarize, we have described the dynamics of two modified Wienbridge oscillators coupled at fixed point. They are synchronized throughout their oscillatory region. The experiment also provides insight to interesting phenomena like the rise of quasiperiodicity and modulation from higher order locking and finally evolving into limit cycle.

3.2. Both Oscillators in Oscillatory Phase

In this section the effect of coupling two oscillators in the oscillatory phase is described. The whole set-up is similar to the previous one. The two oscillators are connected with a 20 kOhm resistor as before, which acts as the coupling parameter. But this time the difference is that both the oscillators are kept at their natural oscillation. There is a huge detuning between the oscillations as their natural frequencies are kept at respectively 90.79Hz and 1139Hz. The detuning is kept for understanding the effect of coupling on the frequencies of the oscillators.

The time series plot of the two channels are shown in Fig.15. The first two plots show the time series plot of channel A and Channel B before coupling and the next two plots show the above after coupling. The change in the waveform in Channel B is clearly visible. The waveform of Channel A effects that of Channel B as its waveform changes into square from nearly sine whereas the waveform of Channel A remains almost same.

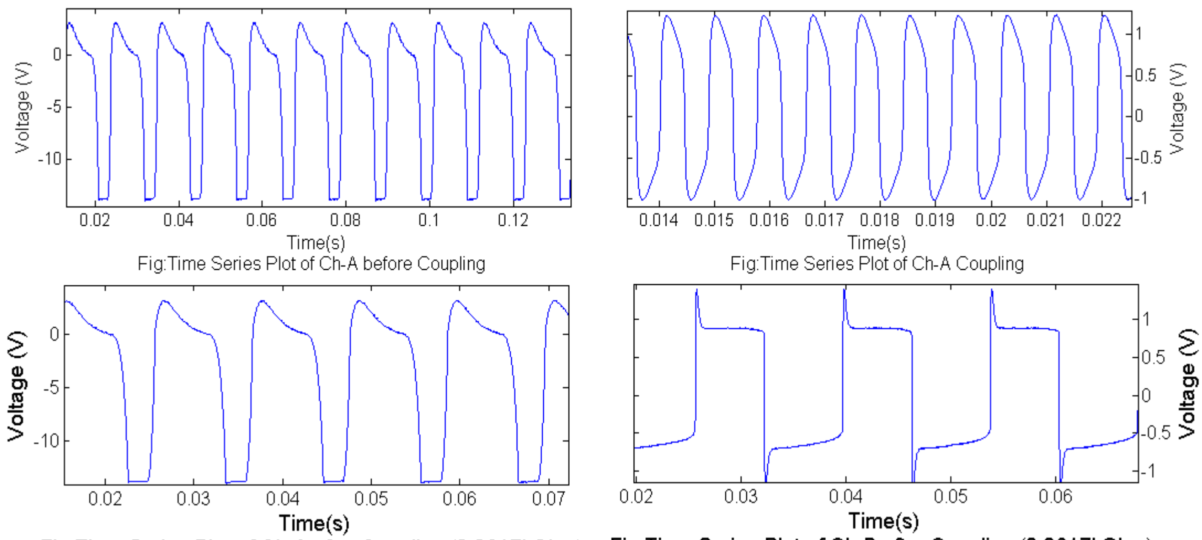

Fig:Time Series Plot of Ch-A before Coupling

Fig:Time Series Plot of Ch-A Coupling

Fig:Time Series Plot of Ch-A after Coupling (0.2617kOhm)

Fig:Time Series Plot of Ch-B after Coupling (0.2617kOhm)

Fig: 15

Not only the coupling effects the output waveform of Channel B it also effects the frequencies of both channel. Just after the coupling the frequencies of the two channel becomes same as it is clear from the FFT plot shown in Fig.16. The data selector shows the frequency of the oscillation of two channels as 48.07Hz. Despite the high initial detuning between the two channels they become synchronized as soon as they are coupled.

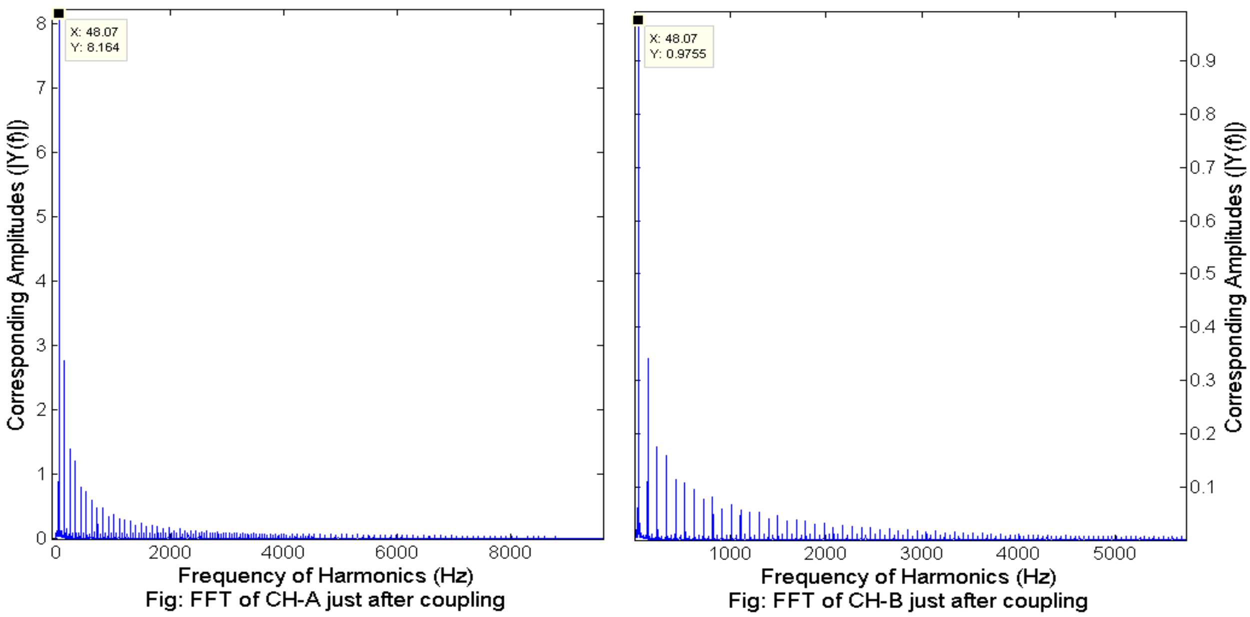

Fig: 16

So the two outputs are synchronized. To establish this fact we plot the frequency of the oscillations of the channels. As we increase the coupling parameter the coupling strength decreases. But the frequencies of their oscillations overlap for the whole region, i.e., they are synchronized for all values of C.P. Fig.17 shows the variation of the frequency against C.P. for two channels. The two frequencies stabilize at nearly 90kOhm which is close to the initial natural frequency of channel 1. This information points to the fact that unlike the coupling at far fixed point where there was mutual dependence between the two oscillators, here the first oscillator (the one with low natural frequency) controls the other one. Oscillator-A acts here as a driving force for Oscillator-B. Due to this driving force Oscillator B becomes synchronized with Oscillator A and oscillates with the natural frequency of A. Hence the frequency of the first harmonic of both the two oscillators gradually reaches to 90Hz.

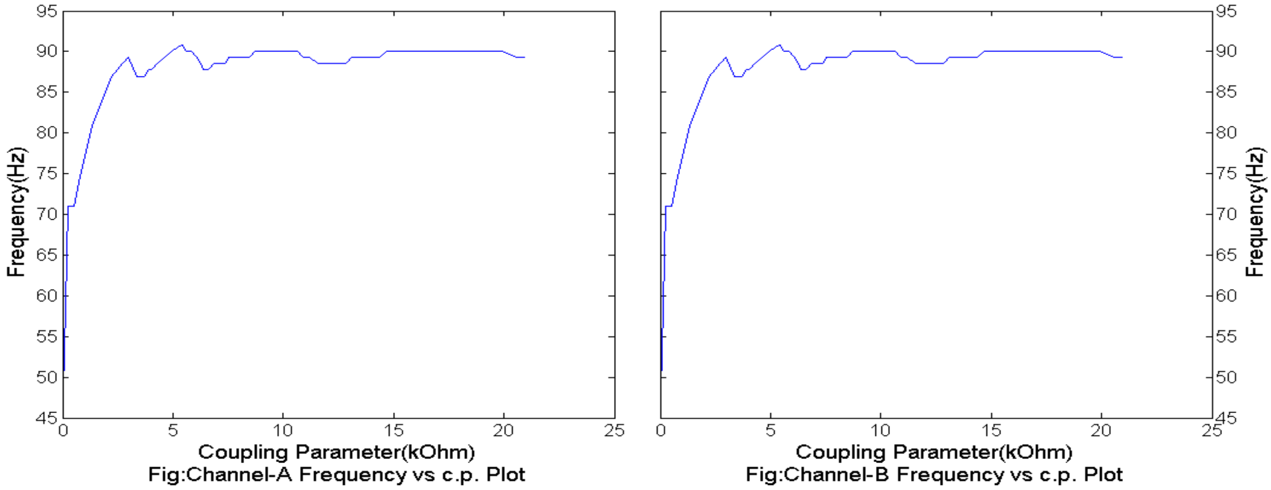

Fig: 17

Though the frequency of the first harmonic of the two channels coincide with each other completely the amplitudes of the oscillations are completely different. The amplitude or the voltage taken as output is greater for Channel 1. The variation of the difference between the amplitudes vs. C.P. is shown in Fig.18 along with the variation of the amplitude of the oscillations with coupling parameter.

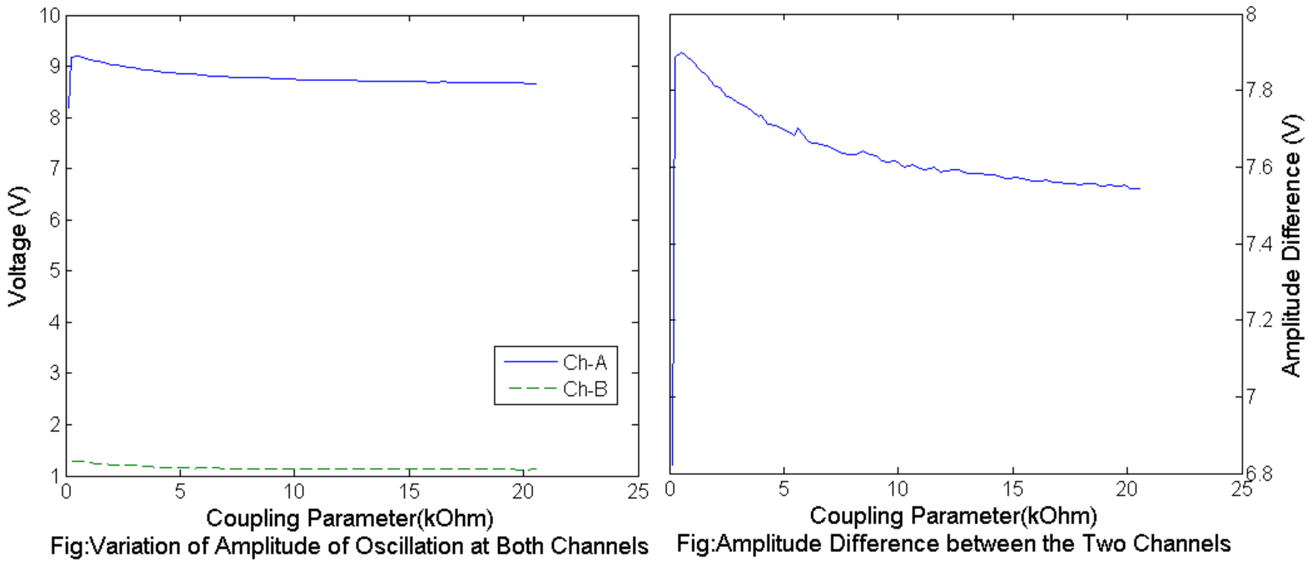

Fig: 18

The variation of the amplitude difference with coupling strength shows that there is no amplitude locking in this case. The difference between the amplitudes of the oscillations gradually decreases after an initial steep rise and tends to get stabilized thereafter.

But there is another interesting thing that must be mentioned in the context of synchronization. That is the rise of a secondary harmonic in channel-B. From the FFT plots, after 3.35 kOhm we can see the rise of three harmonics which was not previously notable. The sudden amplitude rise of these later three frequencies in the FFT plots indicate that there must be some secondary oscillation that contributes to the sudden jump of these harmonics. Whereas in case of channel A there is no significant change in the FFT plots which indicate only one type of oscillation present there. This interpretation is confirmed by the time series plot of channel-B and channel-A shown in Fig.19.

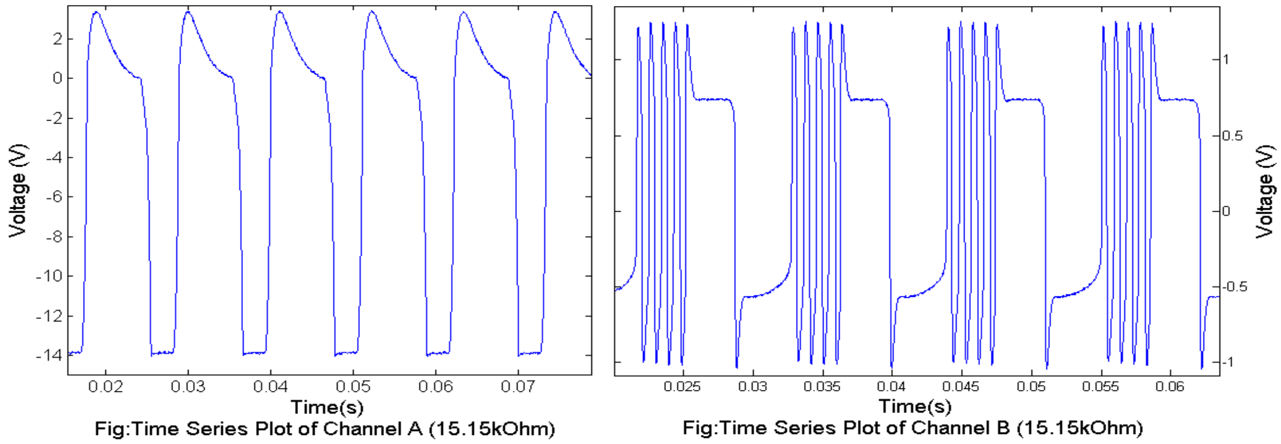

Fig: 19

The time series plot of Channel-A shows no such deviation from that of the initial coupled condition. But the time series plot of Channel-B shows sine waves along with the square wave. This condition arises from coupling resistance 3.35 kOhm where just only one sine wave oscillation is obtained beside a square wave oscillation. So this sine wave oscillation contributes to the secondary harmonic.

This three harmonics have definite domain of frequencies. Their values are respectively near 950 kOhm, 1060 kOhm, and 1140 kOhm. Among these harmonics the third one (near 1140 kOhm) has frequency nearly equal to the natural frequency of the oscillator-B. As the coupling resistance is increased, the coupling strength decreases and the coupled oscillators gradually shifts to their original natural frequency. So, as the coupling resistance is increased the amplitude of the frequency of the third one (near 1140 kOhm) in the FFT plots increases gradually and starts to dominate between the three.

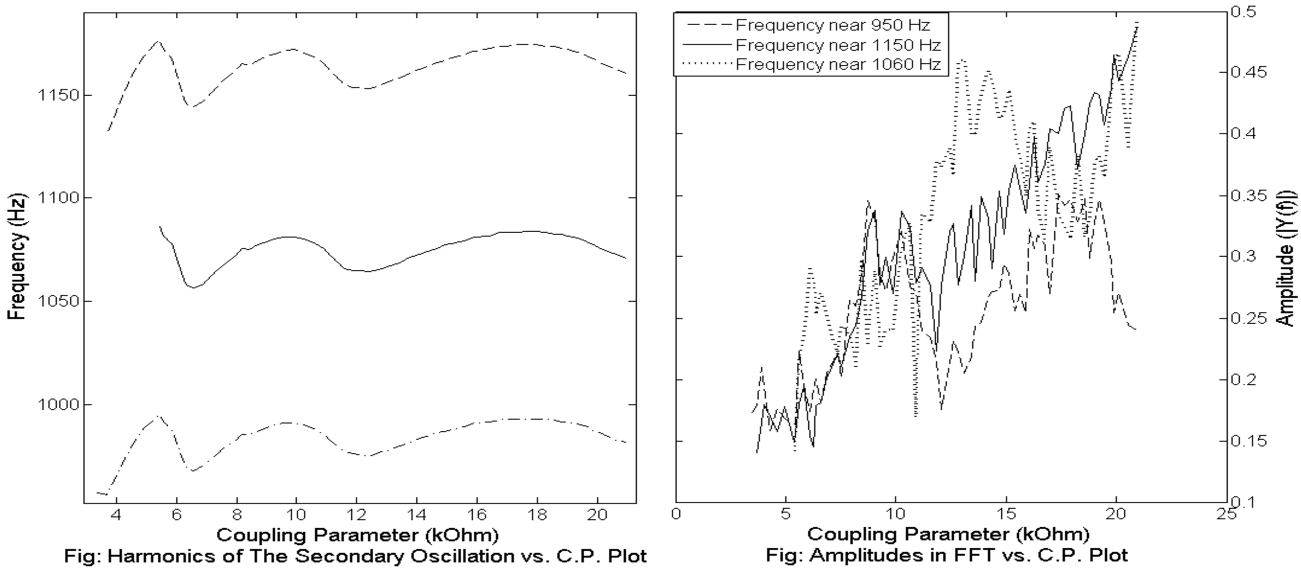

Fig: 20

Fig.20 shows the variation of the frequency of the three harmonics with control parameter. The three harmonics show same kind of variation. The plot in the right of Fig.20 shows the variation of the amplitude of the three harmonics in the FFT plots with coupling parameter. As we can see the amplitude of the third harmonic whose frequency is near 1150Hz is dominant in the end while the second one is dominant in the middle part. So we can conclude that the sine wave frequency or the frequency of the secondary oscillation changes from nearly 1000Hz to 1140 Hz which is the natural frequency of oscillator-B as the coupling parameter is increased.

As before the phase space plots are once more important for the discussion of the dynamics of the coupled system. The phase space plots are given in Fig.21.

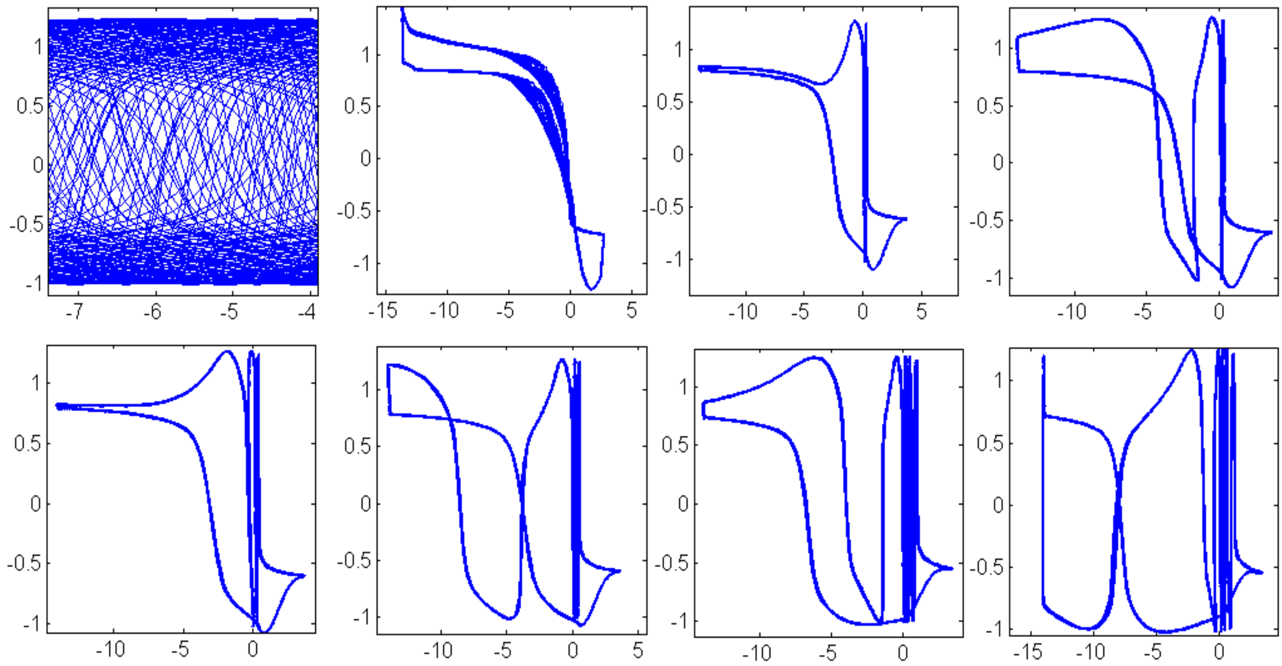

Fig: 21

The figures are arranged in the increased order of C.P. and their respective values are 0 kOhm (uncoupled), 0.0478 kOhm, 5.41 kOhm, 5.58 kOhm, 6.38 kOhm, 8.19 kOhm, 16.25 kOhm, 19.77 kOhm. The first of the figures is the phase space plots of quasi-periodic oscillations as the points occupy the whole phase space region and no two points occupy same place in the plot. This means there is no synchronization. Then just after this plot synchronization occurs as the two circuits are coupled by a variable resistor. The other figure refers to higher order locking. As it can be seen from the time series plots the locking ranges from 2:1 to 6:1. Another interesting phenomenon is that the phase space plots are repetitive in nature as it is evident from the figures. Only the no. of oscillations beside is increased with the increase of C.P. which implies the higher order locking. As the coupling strength decreases the locking order increases.

The oscillations beside the 8 shaped figures have meaning of their own and also provides information about the repetitiveness of the phase space figures. The repetition of a series of figures happen as soon as the order of locking increases from n:1 to (n+1):1. The following set of figures shown in Fig.22 denote such a series as the locking becomes 2:1 to 3:1. The set at the left denotes time series plot of both channels and the right side shows phase space plots (dotted lines indicate Channel A and solid lines indicate Channel B). The first pair of figures are similar to that of the last pair of figures with the exception that the last one has one oscillation more in both the time series and phase space plot.

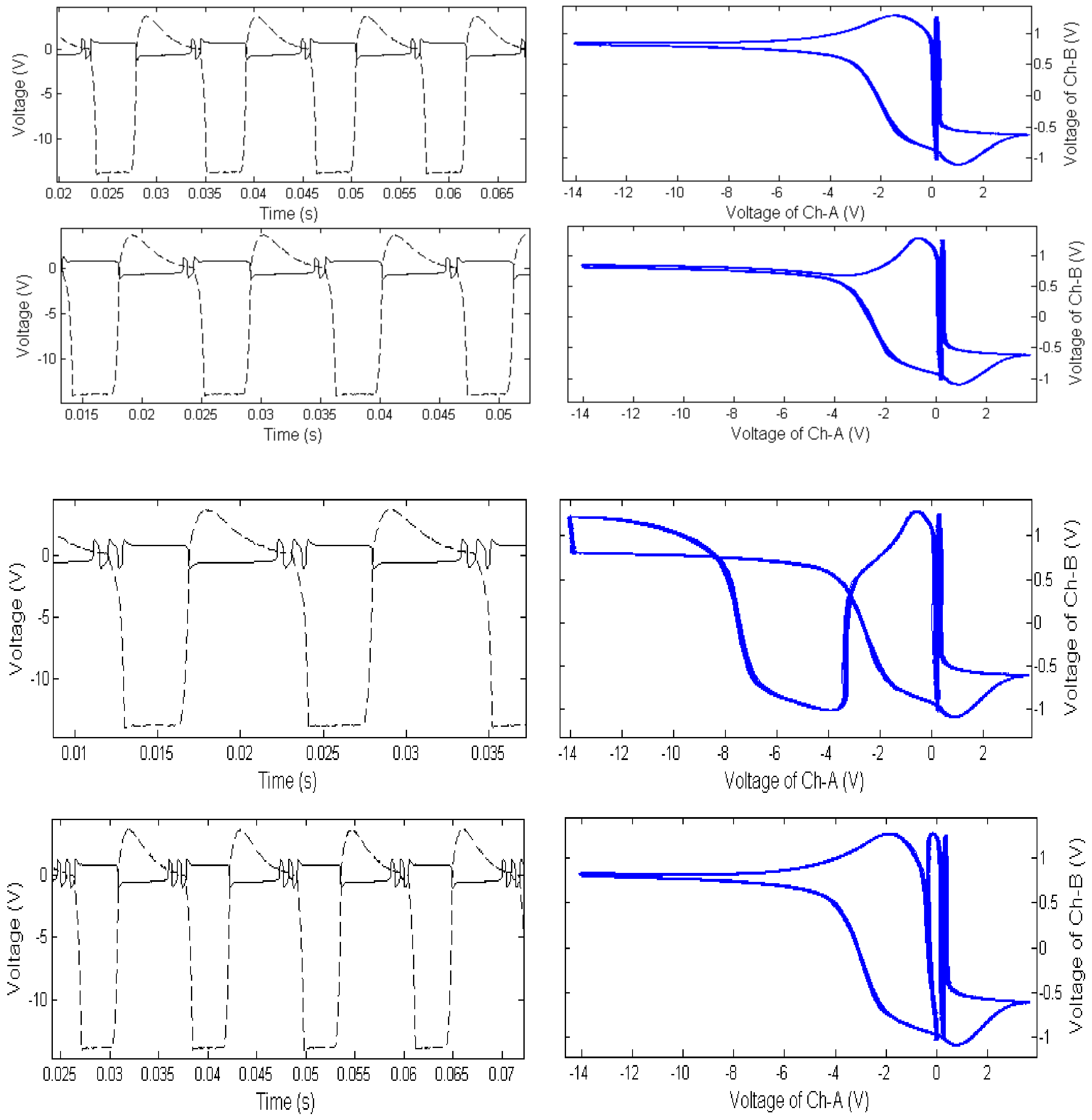

Fig: Time Series and Phase Space Plot

Fig: 22

The corresponding resistance of the time series and phase space plots are respectively 4.28kOhm, 5.43 kOhm, 5.58kOhm, 6.57kOhm. As we can see that the first plot show a 2:1 frequency locking. As the C.P. is increased another sine wave oscillation originates gradually between two square wave oscillations. In the last figure the oscillation created between two peaks enters the region of one oscillation leading to 3:1 locking. This produces one extra oscillation in the phase space plot. And again the repetition happens until another oscillation adds to the side and the order of locking increases.

To summarize, the coupling of two modified Wienbridge oscillators at oscillatory phase gives rise to synchronization. The development of the dynamics is quite interesting as it shows strange phase space plots. Moreover it shades light on relation between higher order frequency locking and the phase space plot.

4. Conclusion

It is shown in the project that coupling between two rather simple systems can produce various strange and interesting phenomena. The dynamics of the coupled systems show rich behaviour like quasi-periodicity and complex Lissajous figures. Also, it is important to observe that to obtain oscillation and synchronization in coupled state, it is not necessary to keep both the oscillators in oscillatory phase. The dynamics show the rise of modulation in oscillations above a critical value of the coupling parameter in the case of coupling the systems at fixed point. The modulating frequency also shows anti-phase synchronization throughout the experiment till both the system reaches fixed point. In the case of coupling two oscillators in oscillatory phase again synchronization appears as well as higher order frequency locking can be observed. The phase space plots show interesting Lissajous figures and quasi-periodic oscillations.

The study of the dynamics of two coupled noise perturbed modified Wienbridge circuit may be the future aspect of this project. This will be important as this coupled circuit can show interesting behaviour when perturbed with noise. We also intend to derive the suitable mathematical model to describe this system. Again the coupling of this oscillator with non-linear chaotic oscillators may give interesting phenomena to which we look forward to.

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6. Index of Figures

Fig.19: *Page 19*, Left: Time Series Plot of Channel A at C.P. = 15.15 kOhm.

Right: Time Series Plot of Channel B at C.P. = 15.15 kOhm.

Fig.20: *Page 19*, Left: Frequency of the Secondary Harmonics vs. C.P. Plot

Right: Amplitudes (Obtained from the FFT Plots) of the Secondary harmonics vs. C.P. Plot

Fig.21: *Page 20*, Phase Space Plots when Both the Oscillators Are at Oscillatory Phase.

Fig.22: *Page 21*, Time Series Plot of both Channels and Their Corresponding Phase Space Plots (Time Series Plots Shown in the Left and Phase Space Plots Shown in the Left).