A brief study on the properties of materials with negative refractive index

Abhijit Chakraborty

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Abstract

We investigate here the consequences of wave propagation within a media of negative permittivity (ϵ) and negative permeability (μ) which is known as metamaterials. The study includes the reverse Doppler effect, Cherenkov effect and study of the effect of evanescent waves in the metamaterials. Further we also discuss the idea behind construction of perfect lens.

1 Introduction

Permeability and permittivity are the two properties of a medium which determines the propagation of electromagnetic waves in the medium. Permeability denotes the degree of magnetisation that a material obtains when external magnetic field is applied. On the other hand permittivity describes how much electric field is generated per unit electric charge in that medium or the resistance that is encountered when forming an electric field in a medium. Generally these two quantities are positive for most materials. But in 1968 Veselago analysed what would happen to the electromagnetic waves if both permeability and permittivity is negative in a medium[1]. Though the whole idea was theoretical, soon it was possible to develop such materials with effective negative permeability and permittivity[2] and experimenting on it. Further this idea gave rise to various new ideas and modifications of the existing ones. People found various applications of the metamaterial in real life because of its interesting qualities[3]. Some of the interesting consequences are discussed in this review including reverse Doppler effect and reverse Cherenkov effect. In 2000 Dr. J. B. Pendry proposed a new idea about making a perfect lens which would not be diffraction limited like the optical lenses we have today[4]. With many people already exploring the modifications in classical electromagnetism, others started working on the quantum domain and proposed theories about how the quantum field theory for electromagnetic field propagation will change for metamaterials[5][6]. Study of metamaterials also led to the idea of a (2+1) Minkowsky space (one of the cartesian co-ordinates is equivalent to time axis) where one can show that an effective black-hole-like horizon appears for electromagnetic waves in a medium at a surface of singular permeability and permittivity[7][8]. Also a solid state analogue of Hawking radiation is proposed using metamaterials[9]. Thus the paper of Veselago opened a whole new branch of Physics. This study, however, is limited to a very small area, mostly on the classical electromagnetic domain. This review includes detailed discussion on how the EM waves behave in metamaterials, and the modificaitons required to our study of refraction and transmission. It also includes a small discussion on the theoretical idea behind perfect lens.

2 Wave propagation in metamaterials

Maxwell's equations in a right handed medium (μ,ϵ>0) can be written as

$$\nabla\times\overrightarrow{E} = -\frac{1}{c}\frac{\partial\overrightarrow{B}}{\partial t}$$
$$\nabla\times\overrightarrow{B} = \frac{\mu\epsilon}{c}\frac{\partial\overrightarrow{E}}{\partial t} \quad (1)$$

Substituting the plane wave solutions

$\overrightarrow{E}= \overrightarrow{E}_0\exp -i(\overrightarrow{k}.\overrightarrow{r}\text{-}\omega t)$ and

$\overrightarrow{B}= \overrightarrow{B}_0\exp -i(\overrightarrow{k}.\overrightarrow{r}\text{-}\omega t)$ in the Maxwell's equations we get,

$$\overrightarrow{k}\times\overrightarrow{E} = \frac{\mu\omega}{c}\overrightarrow{H} \text{ and}$$
$$\overrightarrow{k}\times\overrightarrow{H} = -\frac{\epsilon\omega}{c}\overrightarrow{E} \quad (2)$$

Clearly $\overrightarrow{k}$, $\overrightarrow{E}$, and $\overrightarrow{H}$ construct a right handed system together in the case of a normal material with μ,ϵ>0.

Now we will see what happen to the Maxwell's equations in case of the metamaterials where μ,ϵ<0. We simply substitute μ with - $|\mu|$ and ϵ with -$|\epsilon|$ and rewrite equations (2) as

$$\overrightarrow{k}\times\overrightarrow{E} = -\frac{|\mu|\omega}{c}\overrightarrow{H} \text{ and } \overrightarrow{k}\times\overrightarrow{H} = \frac{|\epsilon|\omega}{c}\overrightarrow{E} \quad (3)$$

which clearly show that now $\overrightarrow{k}$, $\overrightarrow{E}$, and $\overrightarrow{H}$ make a left handed system in contrast with the previous case. It is for this reason the metamaterials are called left handed materials.

Now we know that the refractive index (n) of a material is given by n = $\sqrt{\epsilon\mu}$. So what happen in case of metamaterials? At first look one may say that there should be no change in the refractive index as both ϵ and μhas same sign that leave n unaffected. However the case is not the same. In general the physical quantities ϵ and μ are complex. So we can write $\epsilon= \epsilon_r+ i\epsilon_i$ and $\mu= \mu_r+ i\mu_i$. Consequently n must also be written as $n = n_r+ in_i$.

So we get $n^2 = n_r^2 - n_i^2 + 2in_rn_i$ (4)

and $\epsilon\mu = \epsilon_r\mu_r - \epsilon_i\mu_i + i(\epsilon_r\mu_i + \epsilon_i\mu_r)$ (5)

but the LHS of equations (4) and (5) are equal as n = $\sqrt{\epsilon\mu}$. So the RHS are also same. Equating the imaginary parts of the RHS we get

$2n_rn_i = \epsilon_r\mu_i + \epsilon_i\mu_r$ (6)

But the imaginary parts of ϵ and μ must be always positive because electromagnetic loss in a medium is always postive[10] and in case of metamaterials $\mu_r$and ϵ_r are negative. So overall the RHS of (6) is negative. On the other hand n_i is also positive for the same physical reasons. Hence n_r must be negative. So in an Argand plane all the quantities will be in the second quadrant. This is the reason why we take the refractive index of a left handed substance as negative and we will work with the real part of the refractive index hereafter.

Due to the negative refractive index, the Snell-Descartes Law[11] changes. The refraction at the interface of a right handed media and a Veselago media shows opposite effect. Suppose a monochromatic plane wave $\overrightarrow{E_i}= \overrightarrow{E}_{0i}\exp -i(\overrightarrow{k_i}.\overrightarrow{r}$-$\omega t)$ approaches to the interface (the subscript 'i' denotes incident wave) and gives rise to the following reflected and refracted waves

$$\overrightarrow{E_r}= \overrightarrow{E}_{0r}\exp -i(\overrightarrow{k_r}.\overrightarrow{r}\text{-}\omega t)$$
$$\overrightarrow{E_t}= \overrightarrow{E}_{0t}\exp -i(\overrightarrow{k_t}.\overrightarrow{r}\text{-}\omega t) \qquad (7)$$

The subscripts r and t denote the reflected and the transmitted rays respectively. We also assume that the width of the metamaterial slab is in the z direction (see Fig.1 and Fig.2). Now the wave numbers are related as :

$$k_i v_i = k_r v_r = k_t v_t = \omega$$

Clearly v_r and v_i are equal as they denote the speed of the wave in same medium and v_t is given by $\frac{v_t}{v_i} = \frac{n_1}{|n_2|}$ where n_1 is the refractive index of the right handed medium and n_2 is the refractive index of the left handed media (here we only look at the amplitudeds not directions).

Again using boundary conditions at the interface and equating the exponential parts[11] we get that

$$\overrightarrow{k_i}.\overrightarrow{r} = \overrightarrow{k_r}.\overrightarrow{r} = \overrightarrow{k_t}.\overrightarrow{r}$$

If the wave is propagating in the x-z direction then we get at z = 0,

$$(k_i)_x.x = (k_r)_x.x = (k_t)_x.x \qquad (8)$$

which implies that $k_i\sin(\theta_i)$ = - $k_t\sin(\theta_t)$. The negative sign implies that the wave vector is in opposite direction in the Veselago media as was derived earlier. So we conclude that the modified Snell-Descartes Law for refraction will be

$$\frac{sin(\theta_i)}{sin(\theta_t)} = -\frac{k_t}{k_i} = -\frac{v_i}{v_t} = -\frac{|n_2|}{n_1} \qquad (9)$$

Clearly the angle θ_t is just the negative of the angle that it should have been if the second medium were a right handed medium. The situation is shown in Fig.1 and Fig.2 .

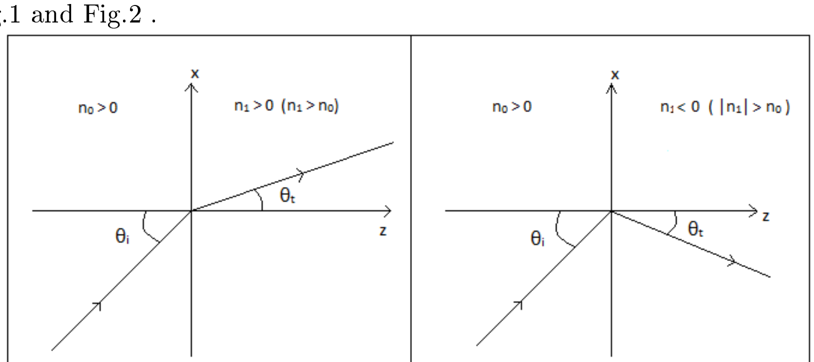

Fig 1 : Refraction in right handed material

Fig 2 : Refraction in left handed material

These are the basics of the wave propagation in a metamaterial. In the following subsections we discuss how TM fields propagate in Veselago medium (metamaterials).

2.1 TM wave propagation in metamaterials

Suppose the plane of incidence is the x-z plane. Then in a right handed system the Maxwell equations for TM harmonic plane waves can be written as (we have taken the convention c = 1 and the time factor $\exp(-i\omega t)$ is implicit)

$$\begin{aligned} &\partial_x H_y = -i\omega\epsilon E_z \\ &\partial_z H_y = i\omega\epsilon E_x \\ &\partial_z E_x - \partial_x E_z = i\omega\mu H_y \end{aligned} \tag{10}$$

These three simultaneous differential equations, when solved, give the following equations

$$\begin{aligned} &E_x = A\sqrt{\mu}cos(\theta)\phi \\ &E_z = -A\sqrt{\mu}sin(\theta)\phi \\ &H_y = A\sqrt{\epsilon}\phi \\ &\phi = \exp i(k_x x + k_z z) \\ &k_x = k\sin(\theta) \\ &k_z = k\cos(\theta) \\ &k = \omega n \end{aligned} \tag{11}$$

If permittivity and permeability are negative as in case of Veselago slabs then in the above expressions [in (11)] just substitute μ with $|\mu|$, θ with θ_1 and $\epsilon$with $|\epsilon|$ and

$$\begin{aligned} &\phi^{*}= \exp -i(k_x^1 x + k_z^1 z) \\ &k_x^1 = k_1\sin(\theta_1) \\ &k_z^1 = k_1\cos(\theta_1) \\ &k_1 = \omega n_1 \end{aligned} \tag{12}$$

We see that ϕ^* is the complex conjugate of ϕ. This is because the direction of $\overrightarrow{k}$ is opposite in the Veselago media.

2.2 Reflected and refracted TM fields within a Veselago media

We rewrite the field equations (11) for normal right handed system as the incident field

$$\begin{aligned} &E_x^i = A^i\sqrt{\mu_0}cos(\theta_i)\phi^i \\ &E_z^i = -A^i\sqrt{\mu_0}sin(\theta_i)\phi^i \\ &H_y^i = A^i\sqrt{\epsilon_0}\phi^i \\ &\phi^i = \exp i(k_x x + k_z z) \\ &k_x = k_0\sin(\theta_i) \\ &k_z = k_0\cos(\theta_i) \\ &k_0 = \omega n_0 \end{aligned} \tag{13}$$

So the reflected fields can be obtained by changing θ_i to $(\pi - \theta_i)$.

$$\begin{aligned} &E_x^r = -A^r\sqrt{\mu_0}cos(\theta_i)\phi^i \\ &E_z^i = -A^r\sqrt{\mu_0}sin(\theta_i)\phi^i \\ &H_y^i = A^r\sqrt{\epsilon_0}\phi^i \\ &\phi^i = \exp i(k_x x + k_z z) \\ &k_x = k_0\sin(\theta_i) \\ &k_z = k_0\cos(\theta_i) \end{aligned} \tag{14}$$

$k_0 = \omega n_0$

Inside the slabs the equations of the transmitted fields can be written easily in analogy with the others. But in this case there will be a contribution from the reflected field from z = d interface (d is the width of the slab, see Fig.3). So the expressions for the electric and magnetic field will be

$$\begin{aligned}
&E_x = [A_1(z) - A_2(z-d)]\sqrt{|\mu_1|}cos(\theta_1)\psi \\
&E_z = -[A_1(z) + A_2(z-d)]\sqrt{|\mu_1|}sin(\theta_1)\psi \\
&H_y = [A_1(z) + A_2(z-d)]\sqrt{|\epsilon_1|}\psi \\
&\psi = \exp -i(k_x x) \\
&A_1(z) = \exp -i(k_z z) \\
&A_1(z-d) = \exp ik_z(z-d) \\
&k_x = k_1 \sin(\theta_1) \\
&k_z = k_1 \cos(\theta_1) \\
&k_1 = \omega|n_1|
\end{aligned} \tag{15}$$

The expressions of the transmitted fields (at z > d) are given by

$$\begin{aligned}
&E_x^t = A^t\sqrt{\mu_0}cos(\theta_t)\phi^t \\
&E_z^t = -A^t\sqrt{\mu_0}sin(\theta_t)\phi^t \\
&H_y^t = A^t\sqrt{\epsilon_0}\phi^t \\
&\phi^t = \exp i[k_0 \sin(\theta_t)x + k_0 \cos(\theta_t)(z-d)] \\
&k_0 = \omega n_0
\end{aligned}$$

The above expressions (13), (14), (15) denote the TM field reflection and refraction by a Veselago slab of width d. Along with appropiate boundary conditions and tedious algebra one can obtain the reflected and transmitted field amplitudes. Fig.3 shows the diagram for reflection refraction and transmission.

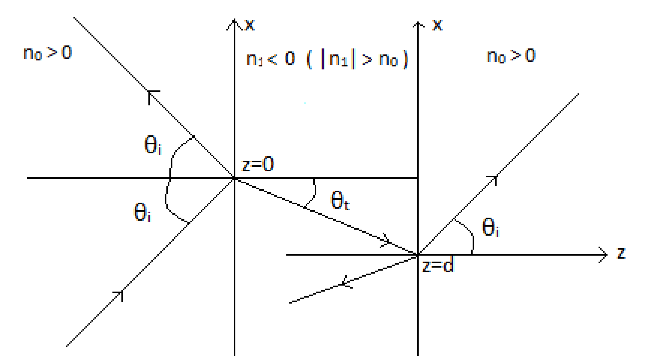

Fig.3 : Propagation of waves through a Vesselago slab of width 'd'.
(Reflection refraction and transmission)

3 Reversal of phase velocity and its consequences

The phase velocity of a wavefront is defined as $\overrightarrow{V_p} = \frac{\omega}{k}\hat{k}$ where $\overrightarrow{k}$ is the wave vector and the Poynting vector is defined as $\overrightarrow{S} = \frac{c}{4\pi}(\overrightarrow{E} \times \overrightarrow{H})$. So $\overrightarrow{S}$ always forms a right handed system with $\overrightarrow{E}$ and $\overrightarrow{H}$. But the wave vector $\overrightarrow{k}$ forms right handed system with $\overrightarrow{E}$ and $\overrightarrow{H}$ in normal medium whereas in metamaterials it forms a left handed system, i.e. in case of metamaterials the direction of wave vector and hence the direction of phase velocity is opposite to the Poynting vector which denotes the direction of the energy flow. This reversal of the direction of phase velocity in left handed systems has some important consequences such as reverse Doppler effect and reverse Cherenkov effect.

3.1 Reverse Doppler effect

If there is relative motion between a source which emits EM wave with a certain frequency and an observer then the frequency the observer experiences, appears different from the original frequency. As an example, if a source moves towards the observer with a velocity ϑ and at the same time emits radiation with phase velocity V_p and frequency f_0 then to the observer the frequency appears to be $f = f_0 \frac{V_p}{V_p - \vartheta}$ which is larger than the original frequency f_0.

But if the waves are moving within a metamaterial then the phase velocity (V_p) has a direction opposite to the Poynting vector $\overrightarrow{S}$. So if we reverse the phase velocity direction and substitute V_p with $-V_p$ in the apparent frequency formula we get

$f = f_0 \frac{V_p}{V_p + \vartheta}$ which is exactly the formula that we would have obtained if the source moved away from the observer in a right handed medium. This apparent frequency is lower than the original frequency. Hence in case of metamaterials we observe reverse Doppler effect. It has recently been possible to demonstrate experimentally.

3.2 Reverse Cherenkov effect

When a charged particle moves through a dielectric medium at a speed greater than its phase velocity at that medium (but less than the speed of light at vaccum, i.e. c), then the electromagnetic radiation emitted by the particle is known as Cherenkov radiation[12]. As the EM radiation cannot move through the medium as fast as the particle (because the EM waves moves at phase velocity of light at that medium where the particle velocity is greater than phase velocity) it produces shock waves as it moves through the medium. A common analogy will be the sonic boom of a supersonic jet.

The emitted radiation creates a forward light cone with half angle θ which is given by the expression

$\cos(\theta) = \frac{c}{n v_p}$ where v_p is the velocity of the particle and n is the refractive index of the medium.

But in case of metamaterials the index of refraction is negative as explained in the previous section. So substituting n with -n in the expression for the half angle of the cone we now get

$\cos(\theta)= -\frac{c}{|n|v_p}$ which indicates that the direction of the cone has reversed. It now produces a backward cone with half angle θ(here $\theta= \pi - \theta'$where $\theta'$was the half angle of the cone produced in the right handed medium).

4 Contribution of evanescent waves in a Vesselago slab

Before starting the discussion on the effect of evanescent waves in a Veselago slab, it is important to know about evanescent waves in normal right handed system. Evanescent waves are formed when waves traveling in a medium undergo total internal reflection at its boundary because they strike it at an angle greater than the so-called critical angle. They should exist because the electric and magnetic field cannot be discontinuous at the boundary. These waves do not propagate and hence carries no enrgy in the direction of propagation. Evanescent waves decay in amplitude exponentially. The mathematical approach about how these waves are generated is discussed below.

Suppose a wave is propagating from a denser medium to a rarer medium. Now from Snell's Law we obtain

$\frac{\sin(\theta_t)}{\sin(\theta_i)} = \frac{n_1}{n_2}$ where $n_2 < n_1$

$$\text{So, } \sin(\theta_t) = \frac{n_1}{n_2}\sin(\theta_i) \tag{16}$$

for $\frac{n_1}{n_2}\sin(\theta_i) > 1$

$\cos(\theta_t) = \sqrt{1-(\frac{n_1}{n_2}\sin\theta_i)^2} = i\sqrt{(\frac{n_1}{n_2}\sin\theta_i)^2-1}$ and we know

$\overrightarrow{E_t}= \overrightarrow{E_0}\exp i(k_1\cos\theta_t z + k_1\sin\theta_t x - \omega t)$

So substituting the value of $\cos(\theta_t)$ we get

$$\overrightarrow{E_t}= \overrightarrow{E_0}\exp(-\alpha z)\exp i(k_1\sin\theta_t x - \omega t) \text{ where } \alpha = k_1\sqrt{(\frac{n_1}{n_2}\sin\theta_i)^2-1} \tag{17}$$

So we see that the evanescent waves are decaying in amplitude along the z direction (which is direction of the width of the slab). If we calculate the Poynting vector and take time average then we see that the waves carry no energy along the z direction. The fact that these waves carry no energy can be shown easily. It is given that [see (13)]

$\overrightarrow{E_t} = [A\sqrt{\mu}\cos(\theta_t)\hat{x}+A\sqrt{\mu}\sin(\theta_t)\hat{z}]\exp(-\alpha z)\exp i(k_1\sin\theta_t x-\omega t)$ using the expressions of $\sin(\theta_t)$and $\cos(\theta_t)$ [see (16) and (17)] and taking the real part of the x component we get

$E_x= -A\frac{\alpha}{k_1}\sqrt{\mu}\exp(-\alpha z)\sin(k_{1x}x\text{-}\omega t)$ where $k_{1x} = k_1\sin(\theta_t)$ and we know that [see (13)]

$\overrightarrow{H} = A\sqrt{\epsilon}\exp(-\alpha z)\exp i(k_{1x}x-\omega t)\hat{y}$. Taking real part of this quantity we get

$H_y = A\sqrt{\epsilon}\exp(-\alpha z)\cos(k_{1x}x-\omega t)$

$$\text{Now we know that the Poynting vector is given by } \overrightarrow{S} = \frac{c}{4\pi}(\overrightarrow{E}\times\overrightarrow{H}) \tag{18}$$

So if we calculate the Poynting vector component in the z direction (the direction of decaying of the evanescent waves) we get

$S_z = \frac{c}{4\pi}(E_x H_y) = -\frac{c}{4\pi k_1} A^2 \alpha \sqrt{\mu\epsilon} \exp(-2\alpha z) \sin(k_{1x}x - \omega t) \cos(k_{1x}x - \omega t)$ (19)

Taking the time average of this quantity we get $\langle S_z \rangle = 0$. So there is no energy propagation in the z direction.

But there is energy propagation in the x direction as :

$\langle S_x \rangle = \frac{c}{8\pi} A^2 \sqrt{\mu\epsilon} \exp(-2\alpha z) \sin(\theta_t)$. So the waves carry energy in the direction perpendicular to its decay.

This is the case of normal right handed systems. So what happens in a metamaterial? It can be very easily shown that the evanescent waves which are decaying in amplitude in right handed systems, become exploding waves with their amplitude growing exponentially in metamaterials. This is demonstrated below.

In a metamaterial we know $\overrightarrow{E_t} = \overrightarrow{A} \exp -i(|k_1| \cos\theta_t z + |k_1| \sin\theta_t x - \omega t)$ [see (12)] where the wave vector has opposite direction to that of the normal right handed medium. Now using the same conditions of the quantities sin (θ_t) and cos (θ_t) as above, we get

$\overrightarrow{E_t} = \overrightarrow{A} \exp(\alpha z) \exp i(k_1 \sin\theta_t x - \omega t)$ where $\alpha = |k_1| \sqrt{(\frac{n_1}{n_2} \sin\theta_i)^2 - 1}$ (20)

So we clearly see that the evanescent wave amplitude increases exponentially within the material. It is also completely consistent with the conservation of energy as these waves carry no energy in the direction of its growth.

5 Perfect Lens

In the above section we saw that evanescent waves does not propagate energy. But then why are the evanescent waves important ? This is because in normal imaging technique we only use the propagating waves and as the evanescent waves are decaying in amplitude they contribute little. In the Fourier expansion of the waves that comes from the substance, we only recieve the propagating part. However in the Fourier expansion there are components also which correspond to the evanescent waves that cannot be captured. So the image resolution is diffraction limited. This could be understood as following

Consider an object and a lens placed along the z-axis so the rays from the object are traveling in the +z direction. The field emanating from the object can be written in terms of its angular spectrum method, as a superposition of plane waves (Fourier series):

E (x,y,z,t) $= \sum_{k_x,k_y} A(k_x, k_y) e^{i(k_z z + k_y y + k_x x - \omega t)}$ where k_z is given by the following relation

$k_z = \sqrt{\frac{\omega^2}{c^2} - \left(k_x^2 + k_y^2\right)}$ (in our previous discussions the k_y component is zero)

All of the components of the angular spectrum of the image for which k_z is real are transmitted and re-focused by an ordinary lens and only these carry energy. However, if

$$k_x^2 + k_y^2 > \frac{\omega^2}{c^2} \tag{21}$$

then k_z becomes imaginary and the wave is an evanescent wave with decaying amplitude. These waves attenuate very fast. As a result it is very hard to recieve these waves optically but these contain information as these are also part of the Fourier expansion. This results in the loss of the high angular frequency components of the wave, which contain information about the high frequency or small scale features of the object being imaged. Hence the highest resolution that can be obtained can be expressed in terms of the wavelength:

$k_{z(max)} \approx \frac{\omega}{c} = \frac{2\pi}{\lambda}$ which is similar to the fact that $\Delta x_{min} \approx \lambda$ and this is roughly the diffraction limit.

But in case of metamaterials the evanescent waves are increased exponentially in amplitude (as shown in the previous section, see(20)). So we can receive the evanescent waves as well using proper thickness of a metamaterial. So they can contribute to the image providing high scale details of the object and hence can help us to constrct a perfect lens (if we take n = -1 then there is no reflection and also the incident wave is completely transmitted and the evanescent waves are also amplified) which is not diffraction limited any more.

6 Conclusion

For the past few years metamaterials have become an extremely exciting research topic. The unique properties of electromagnetic waves that can be observed in metamaterials have attracted considerable attention from the researchers. Negative refractive index materials have shown us possible real life applications also, which can be very fascinating. The prospects of metamaterials include constructing superlens[13], invisible cloak for a object[14], a metamaterial absorber[15] which can be used in photovoltaic cell and so on. Not only on classical domain, due to the unavailability of a strong model of quantum gravity, researchers have proposed analogue gravity models using metamaterials which can predict certain phenomena not explored before. Thus the field of metamaterials is expanding rapidly and we expect to witness and explore more fascinating features of metamaterials in the near future.

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