Probabilistic Analysis of Stochastic Processes

Entropy-based arguments for establishing sample path continuity of general Gaussian processes

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Masters' Project Presentation,
December 7, 2022

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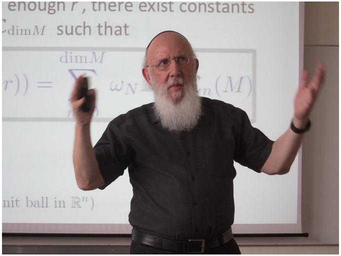

Prof. Robert J Adler

Robert J Adler's Notes on 'An Introduction to Continuity and Extrema of General Gaussian Processes' (1990)

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Motivation for a 'Modern Theory'

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Developments: Many have worked recently on such investigations and developed arguments based on 'metric entropy' and 'majorising measures' for characterizing sample path continuity of Gaussian Processes. Here, we will only be concerned with entropy.

Preliminaries

- $\{X_t | t \in T\}$ such that $\forall t_i \in T, \alpha_{t_i} \in \mathbb{R}$; $\sum_i \alpha_{t_i} X_{t_i}$ is a centered Gaussian random variable, defined on a complete probability space, $(\Omega, \mathscr{F}, \mathcal{P})$.

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- Given t $\in$ T, X is said to be almost sure continuous at t if P($\{\omega \in \Omega | lim_{s-} >_t |X_s(\omega) - X_t(\omega)| = 0\}$)=1.

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What and why?

Definition

Define d:T x T such that d(s,t) $= [E(X_s - X_t)^2]^{(\frac{1}{2})}$. This naturally induces a pseudo-metric in T through the covariance function of X_t, circumventing any dependence on the geometry of T. This d is called the *canonical metric*.

Background

Say X is defined on a (T,τ) and define $\rho_\tau^2(u) = Sup_{\tau(s,t) \leq u} E(X_s - X_t)^2$. Then, for X to be continuous, we atleast need $\rho_\tau^2(u) \longrightarrow 0$ as u $\longrightarrow 0$.
Assuming we have mean square continuity (iff R is continuous), it is observed that for almost sure continuity, only the the rate of convergence of ρ remains the additional concern.

Equivalence of d-continuity and τ-continuity

If X is defined on a compact metric space, (T,τ) with a continuous covariance function R; then τ-continuity and d-continuity of X are equivalent.

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Notice now that $\rho_d^2(u)$ loses information of the covariance matrix, and hence the process itself. We need to tap into the information about the process from d, contained in its relationship with the size and structure of T. How do we go about this?

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The First 'General Theory' Result

It is time to jump to the most important result yet. It's applications and implications for various different Gaussian processes shall be visited now.

Definition

Let N(ϵ) be the least number of d-balls of radius ϵ required to cover (T,d). Then, H(ϵ) = log(N(ϵ) is called the Metric Entropy Function for T (or X).

Note: $N(\epsilon) < \infty \ \forall \ \epsilon > 0$

Main Result

If X is a centered Gaussian Process on a totally bounded parameter space T, and if the integral of the metric entropy over [0,∞), $\int_0^\infty [log(N(\epsilon))]^{\frac{1}{2}} d\epsilon < \infty$; then X is almost surely continuous on T. ....(1)

Remark: The upper limit of the integral can be replaced with diam(T).

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Gaussian White Noise

Definition

Let (E, ε, ν) be a σ-finite measure space. A Gaussian White Noise based on ν is a random set function W on the sets A $\in \varepsilon$ of finite ν-measure such that:
i) W(A) is centered Gaussian and EW$^2(A) = \nu(A) < \infty$
ii) A $\cap$ B $= \phi \implies$ W(A $\cap$ B) $=$ W(A)$+$W(B) a.s.
iii) If A $\cap$ B $= \phi$, then W(A) and W(B) are independent.

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Remarks

1. The covariance function $R_\nu : \varepsilon x \varepsilon \longrightarrow \mathbb{R}^+ \cup \{0\}$ defined as R_ν(A x B) = EW(A)W(B) = ν(A $\cap$ B) , is well defined and positive semi-definite.
2. $d^2(A, B) = \nu(A \Delta B)\ \forall A, B \in \varepsilon$.

Brownian Sheets - What are they and why should one care?

Definition

Let E$=\mathbb{R}_+^k=\{(t_1, t_2, ..., t_k) | t_i's \geq 0\}$, $\varepsilon = \beta_{\mathbb{R}_+^k}$, and $\nu = \lambda$, the Lebesgue Measure. Notate (a, b] $= \Pi_{i=1}^k (a_i, b_i] \subset \mathbb{R}_+^k$ for some $(a_i, b_i]'s \subset \mathbb{R}$. Definitions are similarly extended for (a, b), [a,b] and [a, b). Then the process $\{W_t = W((0, t]) | t \in \mathbb{R}_+^k\}$ is called the Brownian Sheet Process on $\mathbb{R}_+^k$.

Further, the processes defined by $\mathring{W}_t = W_t - |t| W_1$ is called the pinned Brownian sheet. (where $|t| = \Pi_{i=1}^{i=k} t_i$)

- Brownian Sheet is central to the k-dimensional functional central limit theorem. IID processes on the k-dimensional integer lattice in $\mathbb{R}_+^k$ converge weakly to the Brownian Sheet.
- Empirical cdf's and measures built from i.i.d random variables uniformly distributed on $[0, 1]^k$ also converge to the Pinned Brownian Sheet defined over appropriate spaces.

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Continuity of the Brownian Sheet on $[0,1]^k$

Proposition 1

The Brownian Sheet, W_t (and similarly, the pinned Brownian Sheet, $\mathring{W}_t$) is continuous on $[0,1]^k$.

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Proof.

A partition of $[0,1]^k$ is defined such that for each $\epsilon \geq 0$, the function inside the integral of (1) is bounded by a function which has a finite integral over $[0, \infty)$.

Step 1: Define $S_{(t,\delta)} = \{$ s $\in [0,1]^k$: $t_i \leq s_i \leq t_i + \delta$; i=1, 2, 3,..., k$\}$. Prove that $Sup_{s \in S(t,\delta)}$ $[\Pi_{i=1}^{i=k}(t_i + \delta)$ - $\Pi_{i=1}^{i=k}(t_i)] \leq$ kδ. (Trivial for k=1, use induction)

Step 2: Fix $\epsilon > 0$. Then, choosing $\delta \leq \frac{\epsilon^2}{k}$, it can be shown that $S(t,\delta) \subset B_d(t,\epsilon)$.

Step 3: Choose the lattice defined by A= $\{(\frac{i_1\epsilon^2}{k}, \frac{i_2\epsilon^2}{k}, ..., \frac{i_k\epsilon^2}{k}) | i_1, i_2, ..., i_k \in \mathbb{N}_{\lfloor \frac{k}{\epsilon^2} \rfloor}\}$.

Step 4: Show that $[0,1]^k \subset \cup_{t \in A} B_d(t,\epsilon)$. This implies N$(\epsilon) \leq (\lfloor \frac{k}{\epsilon^2} \rfloor + 1)^k$.

Step 5: Bound the integral of $(log(N(\epsilon)))^{\frac{1}{2}}$ over [0,1] by a Riemann Upper sum partitioned in a manner that allows us to obtain a finite bound on the integral. The integral is already finite over $[1,\infty)$.

Hence, from the main result above, W_t is continuous on $[0,1]^k$. □

Brownian Sheet on Lower Layers

Definition

Define a partial order on $\mathbb{R}_+^k$ such that s $\leq t \iff s_i \leq t_i \forall i \in \mathbb{N}_k$ (Similarly extended for '$<$'). Considering $\mathbb{R}_+^k$ fitted with this partial order, a set A $\subset \mathbb{R}^k$ is said to be a lower layer if for any two points, s and t in $\mathbb{R}_k; s \leq t$ and $t \in A \implies s \in$ A.

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Proposition 2

The Brownian Sheet on the lower layers in $[0,1]^2$ is discontinuous and unbounded with probability 1.

Proof.

Idea: Construct two lower layers A and B, such that for any M > 0, W(A) - W(B) $> \frac{M}{2}$, a.e. $\omega \in \Omega$. Then, taking M $\longrightarrow 0$, will give W(A) - W(B) $\longrightarrow \infty$. Hence, it is proved that W is both unbounded and discontinuous over the class of lower layers in $[0,1]^2$.

Construction of the Lower Layers for this particular proof may be shown later if time permits.

What about other indices?

What about other indices?

- Similar proofs also show that W is unbounded over the convex subsets of $[0, 1]^3$. However, W is continuous over convex subsets of the $[0, 1]^2$!.

Example

Example 1 Let $\gamma > 0$. Let A_{01} =[0,1]^2, and $A'_n s$ be the closed rectangles whose left side is the right side of A_{n-1}, so that it has height 1 and width $2^{n(1-\gamma)}$. Further divide each A_n into 2^n equal horizontal slices, $A_{n1}, ..., A_{n2^n}$. Consider this class of sets, $\mathbb{A}_\gamma = \{A_{nj}\}$ and the Gaussian White Noise Process indexed by this class of sets.

Result

It can be shown that $a_1 exp(b_1 \epsilon^{\frac{-2}{\gamma-1}}) \leq N(\epsilon) \leq a_2 exp(b_2 \epsilon^{\frac{-2}{\gamma-1}})$. From Theorem (1) and the above information, we can show that W is continuous on $\mathbb{A}_\gamma$ for $\gamma > 2$; and discontinuous on $\mathbb{A}_\gamma$ for $1 < \gamma \leq 2$

- Clearly, continuity and boundedness depend on the relationships between the process and its parameter space. Regardless, a common tool is handy in analyzing them.

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Processes on $\mathbb{R}$

Proposition 3

Let X be a centered Gaussian Process on a finite interval [0, T]. If for any $\delta > 0$, $\int_0^\delta (-log(u))^{\frac{1}{2}}$ dp(u) $< \infty$, then X is continuous on [0, T].

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Proof.

1. Define p(u) $= [Sup_{|s-t| \leq u} E(X_s - X_t)^2]^{\frac{1}{2}}$. It is easy to observe that diam(T) = p(T). Show that $N_\epsilon = 1 \; \forall \epsilon > 2p(\frac{T}{2})$.
2. Partition [0,T] into $\lfloor \frac{T}{2p^{-1}(\epsilon)} \rfloor + 1$ intervals of length $2p^{-1}(\epsilon)$. 3. Use Theorem (1)'s integral, and obtain an integral of the form in the sufficient condition given, after a change of variables. From the finiteness guaranteed by the sufficient condition, conclude X is continuous on [0,T]. □

Comments -Processes on $\mathbb{R}$

- Avoiding the notion of entropy, and instead relying on the specific geometry of the parameter space in $\mathbb{R}$ confounds the basic issues and leads to a more complicated analysis than the above.
- On the real line, it can be shown that if for some $0 < c < \infty$ and $\alpha, \eta > 0$; $E|X_s - X_t|^2 \leq \frac{c}{|log|s-t||^{1+\alpha}} \forall s, t with |s-t| < \eta$; then X is continuous on [0,T]. This condition actually implies the finiteness of the integral in proposition 3.
- Note that this clearly shows continuity as a consequence of the smoothness of the covariance function at the origin.
- However, as expected from a specialization, some processes are not characterized as continuous directly from the integral in proposition 3.

Gaussian Fourier Series - What and why?

Fourier Series Recall

Any Periodic function, f(t) can be decomposed in the following manner:
$\hat{f}(t) = \sum_{-\infty}^{\infty} c_n e^{int}$

Definition

The sum represented by $\sum_0^\infty a_n Y_n e^{int}$, t $\in [0, 2\pi]$; where,
-$a_n \in \mathbb{R}$ such that -$\sum_0^\infty a_n^2 = 1$,
$\{Y_n\}$ is an iid sequence of random variables such that $Y_n \sim N(0,1)$;
is called a Gaussian Fourier Series.

- The uniform convergence/divergence of series similar to the one described here has a number of consequences in non-random harmonic analysis.
- We will now see how our main result (1) comes handy in proving a theorem important in the context provided above.

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Gaussian Fourier Series - Important Result

Proposition 4

Let $\{Y_n\}$ and $\{Y'_n\}$ be two independent, infinite sequences of independent, standard normal random variables, and $\{a_n\}$ be a non-increasing real sequence. Then the sum $X_t := \sum_0^\infty a_n(Y_n cos(nt) + Y'_n sin(nt)), t \in [0, 2\pi]$, converges uniformly on $[0, 2\pi]$ if, and only if, the following sum converged: $\sum_{j=2}^\infty (\frac{\sum_{n=j}^\infty a_n^2)^{\frac{1}{2}}}{j(logj)^{\frac{1}{2}}}$.

Sketch of reverse side proof

1.Beginning with the convergence of the sum in Proposition 4, it is proven that $\sum a_n^2 < \infty$.
2. For each fixed t $\in [0, 2\pi]$; the Khinchine Kolmogorov 3-Series theorem guarantees the almost sure convergence of $\sum_0^\infty a_n(Y_n cosnt + Y'_n sinnt)$. This implies the existence of the limit process almost everywhere on Ω.
3. A countably dense subset of [0,2π], T is taken and X_t is defined as the limit process on $\Omega - N$, where N is the countable union of Null sets for each t $\in$ T where the limit process may not be defined.

Gaussian Fourier Series -Continued

4. X so defined is then proved continuous on T from Proposition 3. Intermediate steps include the following:
-Finding the covariance function of X_t and deriving from there the value of $p^2(u)$.
- proving the bound $p^2(2^{-n}) \leq 4\sum_{j=0}^{n} \frac{2^{2j}}{2^{2n}} A(2^j, 2^{j+1}) + B(2^n)$ where $A(m,n) = \sum_{j=m+1}^{n} a_j^2$, and B(n)=A(n,$\infty$).
-Using the inequality $(|x|+|y|)^{\frac{1}{2}} \leq |x|^{\frac{1}{2}} + |y|^{\frac{1}{2}}$ and Cauchy Condensation Test to prove the convergence of the above series.
5. Once the above is done, a result that says: If $\int_K^\infty p(e^{-x^2})dx \leq I < \infty \forall K > 0 \iff$ Proposition 3's sufficient condition is satisfied; is used.
6. A result to be studied later is used to conclude that the continuity above implies the absolute convergence of the series required, through a few steps in between.

- Notice how the entropy arguments play an important role in simpler analysis of such processes, with implications in other fields of interest.

The Talagrand Expansion

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Proposition 5

Let X be a centered Gaussian process on a compact metric space T. Then X is continuous $\iff$ X has a continuous covariance function and there exists a centered Gaussian sequence $\{Y_n\}$ with variances $\sigma^2(Y_n)$ such that $\forall t, X_t := \sum_{n=0}^{\infty} \alpha_n(t) Y_n$ with the following conditions satisfied:

- $lim_{n \longrightarrow \infty} (log(n))^{\frac{1}{2}} \sigma(Y_n) = 0$
- For each t $\in$ T, $\alpha_n(t) \geq 0 and \sum_{n=0}^{\infty} \alpha_n(t) \leq 1$.

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- $lim_{n \longrightarrow \infty} (log(n))^{\frac{1}{2}} \sigma(Y_n) = 0$
- For each t $\in$ T, $\alpha_n(t) \geq 0 and \sum_{n=0}^{\infty} \alpha_n(t) \leq 1$.

- This theorem simplifies construction of continuous Gaussian Processes, and tells us that ALL continuous processes can be built by playing with the orthonormal basis for functions on T.
- Not many assumptions on $\{Y_n\}$ and $\{\alpha_n\}$.

Generalised Random Fields

Definition

A stochastic process whose parameter space is either a k-dimensional Euclidean space or a k-dimensional lattice is called a Random Field.

Definition

Let X_t be a centered Gaussian random field on $\mathbb{R}^k$, with covariance function R(s,t). Let $\mathfrak{F}$ be a family of functions on $\mathbb{R}^k$, and for $\phi \in \mathfrak{F}$, define X(ϕ) = $\int_{\mathbb{R}^k} \phi(t)X(t)dt$.
Then, we obtain a centered Gaussian process indexed by functions in $\mathfrak{F}$, whose covariance functional is given by:
$R(\phi, \psi) = EX(\phi)X(\psi) = \int_{\mathbb{R}^k} \int_{\mathbb{R}^k} \phi(s)R(s,t)\psi(t)dsdt$.

- This definition allows us to define function-indexed processes with so defined covariance function as above, even when a point-indexed process with R(s,t) does not exist.
- A whole function indexed process can be defined for any positive definite function R, on $\mathfrak{F}_R = \{\ \phi | \int_{\mathbb{R}^k} \int_{\mathbb{R}^k} \phi(s)R(s,t)\psi(t)dsdt < \infty\}$.

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Generalised Random Fields -Result and Comments

Proposition 6 (stated without proof)

Define $\mathfrak{F}^q(T, C_0, .., C_q)$ where $T \subset_{bdd} \mathbb{R}^k, q > 0$ and $p = \lfloor q \rfloor$ as the class of functions on T whose partial derivatives of orders 0,1,...,p are bounded by finite positive constants $C_0, ..C_p$ and the partial derivatives of order p satisfy Holder conditions of order q-p with constant C_q.
A centered Gaussian process with covariance function,
$R(\phi, \psi) = EX(\phi)X(\psi) = \int_{\mathbb{R}^k} \int_{\mathbb{R}^k} \phi(s)R(s,t)\psi(t)dsdt$ where X_t is a centered Gaussian random field on $\mathbb{R}^k$, with covariance function
R(s,t)$\leq \frac{c}{||s-t||^\alpha} \forall ||s-t|| \leq \delta$, for some $c < \infty$ and $\delta > 0$ will be continuous on $\mathfrak{F}^q(T, C_0, .., C_q) ifk > \alpha$ and $q > \frac{1+\alpha-k}{2}$.

- In most literature, only Schwartz space functions are used for indexing. This is a smaller class of functions. However, the kind stated in the proof is useful in the study of infinite dimensional diffusions, stochasticPDEs, etc.
- The class of generalised fields considered here is also the 'Free Field' used in Euclidean Quantum Field Theory.
- Similar results may be extended to measure-indexed processes.

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Near Future

- There are detailed examples of Set-indexed Processes, Vector-Valued Processes, Banach Space Valued Processes and Non Gaussian Processes.
- The discussion further covers prerequisites required to prove the result (1), and the proof of the same.
- After that, the concept of 'majorising measures' is introduced to characterize continuity and study the distribution of Suprema of Gaussian Processes over a finite set.
- By the end of next semester, I aim to cover all the above, and be equipped with what is needed to understand literature of similar kind.

That's all for now. Thank you for your patience.