A PROJECT REPORT ON

DARK ENERGY

BY

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UNDER THE SUPERVISION OF

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ABSTRACT

This report will introduce the reader to the basics of General Relativity and Cosmology and will also try to explain the theories provided to explain the phenomenon of Dark Energy. This report is aimed at undergraduate students who want to get an overview on this subject. The report is highly non-rigorous in nature. For a mathematical and rigorous approach, kindly refer to the references provided at the end of the report.

Dark energy was hypothesized to explain the accelerated expansion of the Universe. Not much is known about the nature of dark energy. Many issues are open to speculation and debate. There have been attempts to explain dark energy, some of them more successful than others, but none have yet been experimentally verified. It is a window for new physics to arrive.

This report is arranged as follows. In the first chapter I will try to introduce dark energy in very simple terms, deal with its historical origins and also introduce General Relativity and cosmology simultaneously (the currently accepted explanations of the Universe on large scales). Next I will attempt to explain the various theories for explaining dark energy including the cosmological constant, quintessence and chameleon field theories. I will conclude with a short chapter on the experiments being proposed to verify these theories and the work that needs to be done in order to fully comprehend the phenomenon of dark energy.

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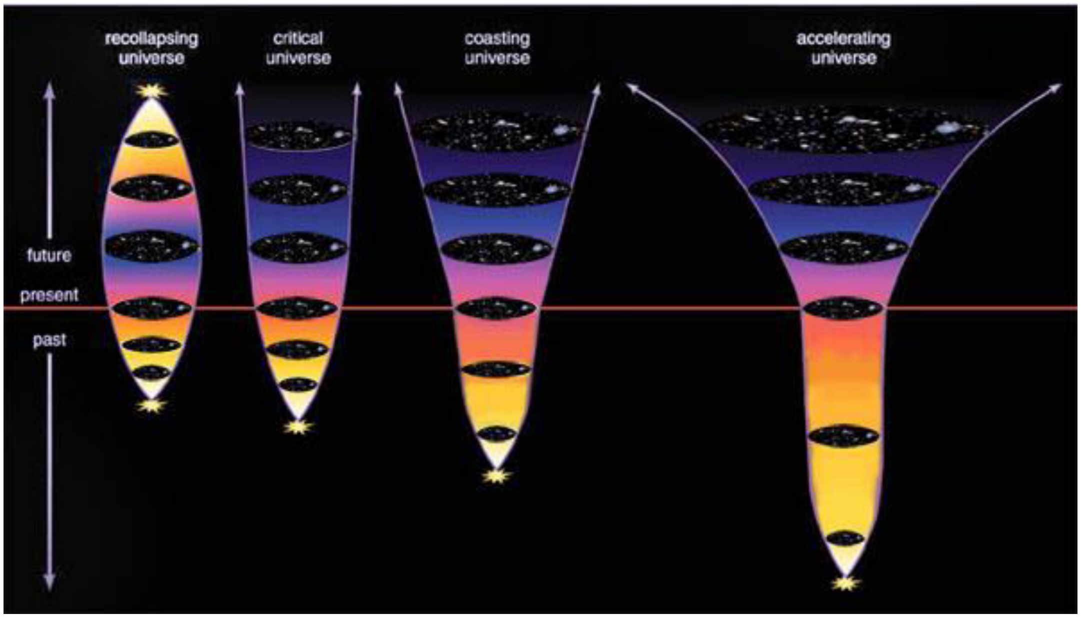
Introduction to Dark Energy

(also General Relativity and Cosmology)

In 1929, American astronomer Edwin Hubble showed that the Universe was expanding as opposed to Einstein’s explanation (who preferred a static Universe). Till the 1980s it was generally agreed that the Universe expanded at an ever slowing rate, since there was gravity to slow the expansion of the Universe. But the question was, just how much was the expansion slowing?

In the 1990s, two independent teams of astrophysicists again turned their eyes to distant supernovae to calculate the deceleration. To their surprise, they found that the expansion of the universe wasn't slowing down, it was speeding up! Something must be counteracting gravity, something which the scientists dubbed "dark energy."

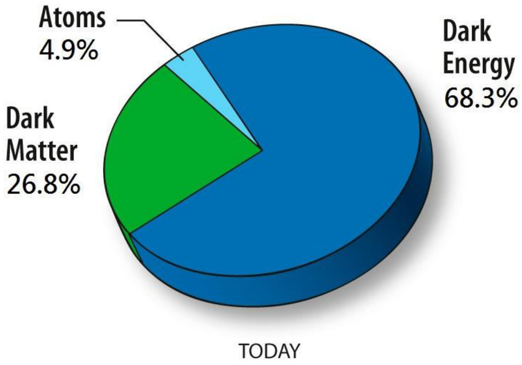


Possible outcomes of the Universe we live in. [1]

So the expansion of the Universe is not slowing down due to gravity, as everyone thought, it has been accelerating. No one expected this, no one knew how to explain it. But something was causing it.

Eventually theorists came up with three sorts of explanations. Maybe it was a result of a long-discarded version of Einstein's theory of gravity, one that contained what was called a "cosmological constant". Maybe there was some strange kind of energy-fluid that filled space. Maybe there is something wrong with Einstein's theory of gravity and a new theory could include some kind of field that creates this cosmic acceleration. Theorists still aren't sure what the correct explanation is.

Calculating the energy needed to overcome gravity, scientists determined that dark energy makes up roughly 68 percent of the universe. Dark matter makes up another 27 percent, leaving the "normal" matter that we are familiar with to make up less than 5 percent of the cosmos around us.



Brief Introduction to General Theory of Relativity

Einstein's general theory of relativity represents our most fundamental understanding of space, time and gravitation. It was published by Albert Einstein in 1916 in order to find a geometric theory of gravitation, and is today the accepted description of gravity in modern physics. The theory is a unification of special relativity and Newton's law of gravity, and describes gravity as a property of the geometry of spacetime. I will not go into a detailed analysis of the theory. I will just present an overview of the theory for the uninitiated to understand the essence of the theory.

General relativity is a tensor-based theory of gravitation. It describes gravity as the curvature of spacetime instead of a force. In short, Einstein says that matter curves "spacetime" which in turn tells other massive particles how to move. It is valid in all reference frames (since it is described by tensor equations).

What is a tensor?

Tensors are simply mathematical objects that can be used to describe physical properties, just like scalars and vectors. In fact tensors are merely a generalization of scalars and vectors; a scalar is a zero rank tensor, and a vector is a first rank tensor. You may think of tensors as 2-dimensional matrices. By that analogy, vectors would be a matrix with a single column and scalar just a single number.

Einstein's theory involves two tensors. One is the Einstein tensor and the other is the stress-energy-momentum tensor. According to Einstein those two are proportional. In other words,

$$\boxed{G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}}$$

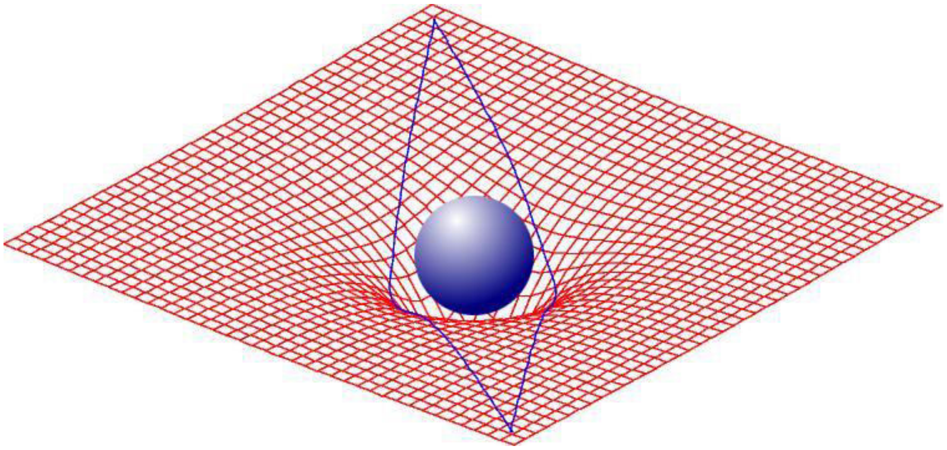
where G is the gravitational constant, c is speed of light, $G_{\mu\nu}$ is the Einstein tensor, $T_{\mu\nu}$ is the stress-energy-momentum tensor and μ and ν are indices running from 0 to 3 indicating 4 dimensions of spacetime. So, in principle there are 16 equations in the above formula, but 6 of them are redundant. So that makes for 10 equations. These equations are called Einstein's Field Equations. In very broad terms, $G_{\mu\nu}$ is related to the curvature of spacetime and $T_{\mu\nu}$ is related to the mass/energy which dictates how spacetime curves.

The Einstein tensor itself is composed of two tensors.

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu},$$

where $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar and $g_{\mu\nu}$ is the metric tensor. More details on these tensors and the theory in general can be found in the references.

The metric tensor $g_{\mu\nu}$ is a tensor which depends on the kind of geometry we are considering. For a flat spacetime (Minkowski spacetime) the metric tensor $g_{\mu\nu} = \eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$ (This is just a convention, diag(1,-1,-1,-1) can also be used)



Matter curves spacetime and other objects move on the shortest path possible in the curved space (which are called geodesics). Geodesics are basically the shortest paths between two points in curved spacetime. That is how light bends around massive objects.

Basic Cosmology

We can use the Einstein Field Equations to solve for the evolution of the Universe. This can be done by putting in different metric tensors ($g_{\mu\nu}$) or stress energy momentum tensors ($T_{\mu\nu}$). But for solving for the Universe as a whole, certain simplifying assumptions are made (which are backed up by experimental evidence). The basic assumption made is the Cosmological Principle which says that at large enough scales, the Universe is homogenous (translational invariant) and isotropic (rotational invariant). Of course that assumption may not seem true on the Solar System level, but it is true on the level of galaxy clusters and beyond. Also, the Universe can be approximated by a perfect fluid with a certain energy density ρ and pressure p.

As stated above, different metric tensors can give different models of the Universe as solutions. The metric we are going to use here is the Friedmann-Lemaitre-Robertson-Walker metric. It describes a homogenous, isotropic expanding Universe. The line element (square of distance between two points) is given by,

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

where a(t) is the scale factor and k is the curvature parameter. If k=0 then space is flat. Current observations indicate that space is indeed flat (so k is close to 0). The metric tensor is given by $g_{\mu\nu}=\mathrm{diag}\,(-1,a^2(t), a^2(t), a^2(t))$ and the stress energy momentum tensor is $T_{\mu\nu} = \mathrm{diag}\,(-\rho,p,p,p)$ (which is the stress energy momentum tensor for a perfect fluid)

Plugging both of them into Einstein's Field Equations yields two Friedmann Equations.

1st Friedmann Equation - $$\frac{\dot{a}^2 + k}{a^2} = \frac{8\pi G}{3}\rho,$$

2nd Friedmann Equation - $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

We need two more equations for solving the equations. One comes from conservation of energy and momentum

$$\dot{\rho} + 3H(\rho + p) = 0$$

Where H is the Hubble parameter ($H=\dot{a}/a$). Last equation is the equation of state $p=w\rho$ where w is the equation of state parameter (w=0 for matter, w=1/3 for radiation and w=-1 for dark energy). Solving the above four equations, we can determine ρ (energy density) and a(t) (scale factor) as functions of time.

For example -

- $a(t) \propto t^{1/2}$ during radiation domination (w=1/3)
- $a(t) \propto t^{2/3}$ during matter domination (w=0)
- $a(t) \propto \exp(t)$ for dark energy dominated era. (w=-1)

I have implicitly used the cosmological constant here for calculating the scale factor for dark energy dominated era. I will expand more on that in the next chapter.

Theories for Dark Energy

Cosmological Constant

As stated previously, Einstein had preferred a static Universe which means that he had to introduce a non-zero cosmological constant into the equations. That was because of the following reason. Einstein considered the Universe to be matter dominated which implies that the pressure $p=0$. Also $a=$constant, since the Universe does not expand. From the second Friedmann equation,

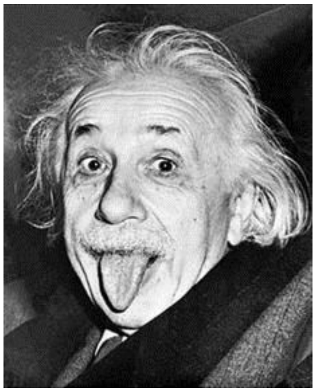
$$0 = 4\pi G\rho/3$$

this implies that $\rho=0$. The Universe is empty! To avoid this paradox, Einstein introduced what is known as the cosmological constant into his equations. The cosmological constant is denoted by Λ. The cosmological constant has $w=-1$. It has negative pressure. If the cosmological constant term dominates, then the Universe's expansion will be accelerated (can be inferred from the second Friedmann equation).

After Hubble discovered that the Universe is expanding, Einstein dismissed the cosmological constant. Even though Einstein called it the biggest blunder of his life, the cosmological constant has come back into the equations because it was discovered that the Universe is actually accelerating. When we put $w=-1$ in the Friedmann equations and solve then we get a model whose expansion is accelerating (it may not be used to describe the entire evolution of the Universe but it may be applicable today as we have accelerated expansion today).

Physically, the cosmological constant is equivalent to vacuum energy. This means that even empty space has some energy. As the Universe expands, more and more space comes within the cosmological horizon => more vacuum energy => space expands faster.

The cosmological constant is a part of the leading cosmological model used today to explain the Universe, i.e., the ΛCDM model. But cosmological constant cannot fully explain dark energy. There are a few problems reconciling the cosmological constant with particle physics.



Cosmological constant is vacuum energy. Even quantum field theories predict that vacuum has some energy. The simplest estimates of vacuum energy from quantum theory give a value that is off by $\mathbf{10^{120}}$ times the observed value. This phenomenon is known as the vacuum catastrophe.

There is another problem with the cosmological constant. It is called the coincidence problem. In layman's terms, it asks the question that why is it that the matter energy density and vacuum energy density is of the same order today? Their ratio must have been a specific value in the early Universe, for them to coincide now. If the ratio would have been different (suppose dark energy was dominant earlier than today), then it would have ripped apart the Universe and there would have been no formation of stars and galaxies and planets (and no human life).

Quintessence

Taking into consideration the problems related to the cosmological constant, scalar fields were introduced. Cosmological constant does not vary in space or time. By introducing the scalar field we can make the cosmological constant dynamical. Scalar field couples to matter, but it is very light so that it does not clump and form structure. It also self-interacts. It also has a tracker behavior (it is insensitive to initial conditions) which helps it evade the coincidence problem.

The action of the scalar field looks like,

$$S = -\int(\tfrac{1}{2}\, g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi))\, \sqrt{(-g)}\, d^4x$$

From the Euler-Lagrange equation, we deduce that (for a spatially homogenous field),

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0,$$

Now using the above action and the stress energy momentum tensor, we can derive the pressure and energy density.

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi),$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

The equation of state parameter w hence becomes,

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$

The potential V(ϕ) can be constructed so as to fit observations. In the limit V(ϕ) much greater than the kinetic energy term, we get w≈-1 (this is known as the slow roll approximation). This also means that the potential is moving slowly.

Although quintessence models seem to describe dark energy very well, there are problems associated with it. One of the major problems with quintessence is that the scalar fields couple to matter. This would lead to a fifth force. That would lead to violations of the Equivalence Principle. Such violations have not yet been detected in local tests of gravity. This puts very strict constraints on the scalar fields which forces the gravitational coupling to be very small or the interaction range to be very short. To overcome such difficulties, the chameleon field theory has been proposed which is described in the next section.

Chameleon Field Theories

To avoid violations of equivalence principle, scalar field to matter coupling should be extremely small. Chameleon field possesses that property. Chameleon field is a field which has a mass that depends on the background matter density. So, in principle, the areas where there is high density of matter, the chameleon field is short-ranged, since the mass of the mediator particle is large. This property would help in avoiding violations of the equivalence principle at small scales. At large scales, where the matter density is low, the chameleon field is long-ranged, since the mass of the mediator particle is small.

The action for the scalar field is given as,

$$S = \int d^4x\sqrt{-g}\left|\frac{M_{Pl}^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right| + \int d^4x\mathcal{L}_M(\psi^{(i)}, g_{\mu\nu}^{(i)})$$

This results in the chameleon equation of motion,

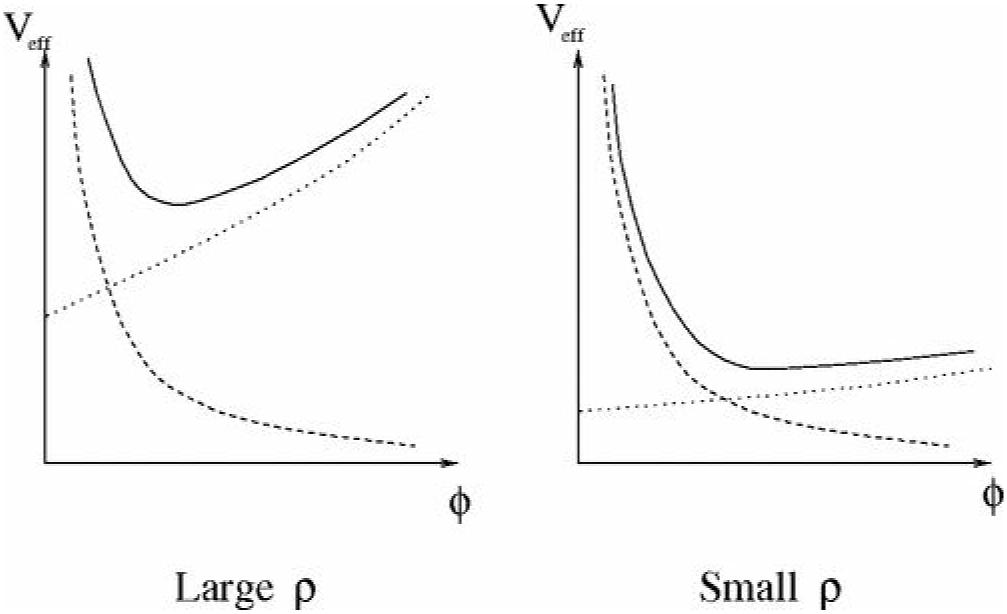
$$\Box\phi = V_{,\phi}(\phi) - \frac{\beta}{M_{Pl}} e^{4\beta\phi/M_{Pl}} \tilde{g}^{\mu\nu}\tilde{T}_{\mu\nu}$$

If the equation is closely observed, we see that the potential has been made dependent on the background matter density. V has been replaced by V_{eff}.

$$V_{eff} = V(\phi) + \rho e^{\beta\phi}$$

where ρ is the background matter density. The mass of the force carriers is dependent on the effective potential as

$$m^2 = d^2V_{eff}/d\phi^2$$



For large ρ, mass of the mediator particle is high => range is small and vice versa.

Future Prospects

Even with such theoretical advancement, we need experimental evidence to prove the theories right or wrong. Many experiments are in development stage or already working to solve the dark energy mystery. A few examples are given below.

Satellite Test of Equivalence Principle (STEP)

STEP will test the validity of the Equivalence Principle. The Equivalence Principle basically states that the inertial and the gravitational mass are the same. This postulate cannot be proven, it can only be tested to higher and higher precision. STEP will be able to test the EP to a sensitivity at least five orders of magnitude better than currently achievable (about 1 part in 10^{13}). If any violation in EP is detected, then that would be a good check for the chameleon theories which do predict EP violations at that level. “MICROSCOPE” and “Galileo Galilei (GG)” are two other satellites which are working on the same principle.

GammeV experiment

Chameleon field particles also couple to photons. Photons and chameleon particles can oscillate between each other in the presence of an external magnetic field. Chameleons can also be confined in hollow containers because their mass increases as they penetrate the container wall, causing them to reflect. This is the strategy used in GammeV experiment. Photons are directed into a cavity, confining the chameleons produced. Then the light source is switched off. The chameleons produced decay back into photons producing an afterglow which is then detected. The latest results were published in November 2010 and nothing significant was found in the testing limits although they were able to put constraints on photon-chameleon coupling.

If chameleon field (or any other scalar field for that matter) exists, then it would affect all gravitational interactions within this Universe. If we could detect the change in gravitational interactions (in the presence of scalar/chameleon fields), then it would help in confirming or refuting any scalar field theory. In the long term that is the aim of our group. Each of us would work on a specific gravitational phenomenon and observe how it changes in modified gravity theories and then test whether the predictions from the modified theories are correct or wrong. I will be investigating the influence of modified gravity theories on gravitational waves. And hopefully, I will be able to prove Einstein's general theory of relativity wrong and win a Nobel Prize. Okay, that was a bit too much but nevertheless it is possible. Who knows someday we may find a Grand Unified Theory which may explain everything that goes on in our Universe. Until then, stay tuned.

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