Quantum Computation using Continuous Variables

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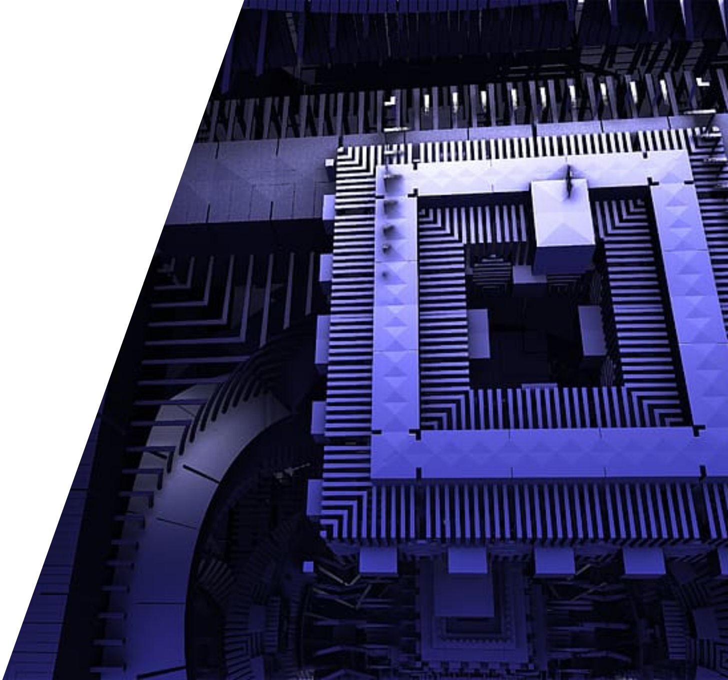

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Introduction

- The article discusses about Quantum Computation using continuous variables like Position or momentum(Quadratures of EM wave) as opposed to traditional discrete variables like spins(Qubits).
- Discusses differences or pros and cons of Continuous and Discrete Variables
- It generalises the results of discrete variables for continuous variables.
- Introduces some protocols for Quantum Computation like QKD,Entanglement distillation in CV setting.

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Requirements for Quantum Computation

- Preparation of highly quantum states like Entangled pairs(Gaussian two mode squeezed state).
- Fabricating Unitary gates for state manipulation(Beam Splitter,Phase Shifter).
- Measurement of States(Homodyne Detection).
- Storage of Quantum information or Quantum memory.
- Transmission of states using Quantum Channels(Photons) securely(Cryptography) and safely(QECC).

Differences between CV and DV

Continuous Variable

Easy to obtain Entanglement

Unconditional preparation

Degree of entanglement is low

Less prone to noise.

Discrete Variable

Hard to obtain Entanglement

Conditional preparation i.e depends on measurement results.

Degree of entanglement is high

More prone to noise.

Continuous variables

- Continuous variable refers to Degrees of Freedom of a system that can take continuous values like position and momentum, as opposed to spin which can take discrete values.
- We use bosonic sysyems like photons to use its quadratures to represent quantum information.
- Hamiltonian of light in a cavity can be written as $\frac{1}{2}\sum \hat{p}_k^2 + \omega_k^2 \hat{x}_k^2$
- This means it is analogous to system of independent harmonic oscillators of different modes.
- We can write $\hat{x}_k = \hat{a}_k + \hat{a}^\dagger$ and $\hat{p}_k = i(\hat{a}_k - \hat{a}^\dagger)$, the ladder operators of corresponding harmonic oscillator mode k.
- Qaudratures of an electromagnetic field are the dimensionless position and momentum operators.

Homodyne Detection

- Homodyne Detection is measurement of Quadratures
- Electric field can be written after second quantization as.
- $\hat{E} = E_0[^{\theta}_{k} cos(w_k t - \mathbf{K.r} - \theta) + \hat{p}^{\theta}_{k} sin(w_k t - \mathbf{K.r} - \theta)]$
- X^{θ} is related to X by rotation matrix of parameter θ.
- $\hat{X}^{\theta} = e^{-i\theta}\hat{a}_k + e^{i\theta}\hat{a}^{\dagger}_k \quad , \quad \hat{P}^{\theta} = e^{-i\theta}\hat{a}_k + e^{i}\hat{a}^{\dagger}_k$
- We couple photons whose quadrature we need to measure with local harmonic oscillator in coherent state with high amplitude $\hat{a}_{L.O} \sim \alpha_{L.O}$
- $\hat{a}_1 = \hat{a} + \alpha_{L.O}$,$\hat{a}_2 = \hat{a} - \alpha_{L.O} \implies \delta i = i_1 - i_2 = q(\alpha^*\hat{a} + \alpha\hat{a}^{\dagger})$

Phase Space Representations

- There is an isomorphism between Q.M states and Quasi-Probability distributions in phase space variables like x,p.
- Wigner function is one such representations with following properties.

W(x,p)=$\int dy e^{4iyp} \langle x-y| \hat{\rho} |x+y\rangle$ (Wigner in terms of transition probability)

$\int W(\alpha) d^2\alpha = 1.$ (Sum of probabilities is 1)

$\int W(x,p)dx = \langle p| \rho |p\rangle.$ (Marginal dist. of x gives position wavefunction)

$\int W(x,p)dp = \langle x| \rho |x\rangle.$ (Marginal dist. of p gives momentum wavefunction)

$Tr[\hat{\rho} S(\hat{x}^n, \hat{p}^m)] = \int W(x,p) x^n p^m dxdp$ (Weyl Correspondence)

More on Quasi Distributions

- There is a family of such Quasi probability distributions.
- $P(\alpha, s) = \frac{1}{\pi^2} \int \eta(\beta, s) exp(i\beta\alpha^* + i\beta^*\alpha) d^2\beta$.
- $\eta(\beta, s) = Tr[\hat{\rho} exp(-i\beta\hat{a}^\dagger - i\beta^*\hat{a})] exp(s|\beta|^2))$
- Here for s=-1 is Q functions and s=1 the P functions used to calculate mean of operators of normally ordered and anti-normally ordered combinations of $\hat{a}, \hat{a}^\dagger$
- Negative probability of Wigner function is a smoking gun for Quantumness in Experiment.
- Used to design states for Quantum Communication.

Gaussian States

- Are efficiently produce-able in labs and also available unconditionally.
- Gaussian States have gaussian wigner distribution and completely determined by first and second moments through correlation matrix.
- The wigner correlation matrix has following properties.

$$\langle(\hat{\xi}_i\hat{\xi}_j + \hat{\xi}_j\hat{\xi}_i)/2\rangle = \int W(\alpha)\xi_i\xi_j d^{2N}\xi = V^N_{ij}, \hat{\xi} = (x_0, p_0, x_1, p_1, \ldots\ldots x_n, p_n)$$

Real,symmetric,positive matrix $\leftrightarrow$ Correlation State of physical state.

$$[\hat{\xi}_i, \hat{\xi}_j] = \frac{1}{2}\Lambda_{ij}, \implies V^N - \frac{1}{4}\lambda \geqslant 0.$$

For gaussian states this condition is also sufficient to prove positivity of state.(imp. in separability criterion)

Linear Optics

- Deals with phenomenon where there is linear mixing of optical modes.
- Conservation of photon number or intensity means its unitary.
- Any linear optics can be decomposed into phase shifters , beam splitters.
- Not all Unitary evolution can be modelled by linear optics.

$$\begin{bmatrix} a'_1 \\ a'_2 \\ \vdots \\ a'_m \end{bmatrix} = \mathbf{U}_{M*M} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$$

$$\psi(x_1, x_2) \to \psi(x'_1 sin(\theta) + x'_2 cos(\theta), x'_1 cos(\theta) - x'_2 sin(\theta))$$

Non-Linear Optics

- Deals with phenomenon beyond linear mixing of optical modes.
- Also includes linear mixing of creation and annihilation matrix.
- Most general is Bogolibuov transformation.

$$\hat{a}'_i = \int_j A_{ij}\hat{a}_j + B_{ij}\hat{a}^\dagger_j + \gamma_i, : AB^T = (AB^T)^T, AA^\dagger = BB^\dagger + I \ \ (Bogolibuov)$$

$(x'_1, x'_2) \rightarrow M(x_1, x_2)$,, M need not preserve inner product.

Parametric down conversion / Squeezing

- Squeezing refers to reductions of the variance of one quadrature while increasing the variance of its conjugate.
- Useful to encode information in reduced quadrature.
- We produce squeezed state through parametric down conversion, parametrized by (r, θ).
- r refers to degree of squeezing and θ refers to angle of squeezing.
- Here, a signal of low intensity(also vacuum) is squeezed and also amplified by an intense signal of double frequency.

Single mode Squeezing

H$_{int} \propto \hat{a}^{\dagger 2}\hat{a}_{pump} - a^2 a_{pump}$ (interaction hamiltonian for parametric amplification)

$H_{int} = i\frac{\hbar\kappa}{2}(\hat{a}^{\dagger 2}e^{i\theta} - a^2 e^{i\theta})$: $(a_{pump} \propto \alpha)$ for intense pumper

$\hat{a}(t) = \hat{a}(0)cosh(kt) + \hat{a}^{\dagger}(0)sinh(kt)$ Heisenberg evolution

$\hat{x}(t) = e^{kt}\hat{x}(0)$ p(t)=e$^{-kt}\hat{p}(0)$ Squeezing of qudratures for $\theta = 0$

U = $exp[\frac{\kappa}{2}(\hat{a}^{\dagger 2}e^{i\theta} - a^2 e^{i\theta})]$ $\hat{S}^{\dagger}(\gamma) = exp(\gamma^{*}\hat{a}^2 - \gamma\hat{a}^{\dagger 2})$ (General Squeezing operator)

$\psi(x) = (\frac{2}{\pi})^{1/4}e^(r/2)exp[-e^(2r)(x - x_{\alpha})^2 + 2ip_{\alpha}x - ix_{\alpha}p_{\alpha}]$ α is displacement parameter, ψ is squeezed displaced state.

$W(x, p) = \frac{2}{\pi}exp[-2e^{+2r}(x - x_{\alpha})^2 - 2e^{-2r}(p - p_{\alpha})^2]$

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Entanglement in Continuous Variables

- Einstein Originally considered entanglement in Continuous D.O.F like position and momentum of two particles.

$$\int dx_1 dx_2 \psi(x_1, x_2) \left|x_1, x_2\right\rangle \propto \int \left|x_1\right\rangle \left|x_1 - u\right\rangle \quad : \quad \delta(x_1 - x_2 - u), \delta(p_1 + p_2)$$

- This is nonphysical, so we consider gaussian two mode squeezed states
- multi mode squeezed gaussian states reach EPR state when squeezing tends to infinity,()

$$\psi(x_1, x_2) = N\, exp[-e^{-2r}(x_1 + x_2)^2/2 - e^{+2r}(x_1 - x_2)^2/2]$$

$$W = N'\, exp[-e^{-2r}(x_1 + x_2)^2 + (p_1 + p_2)^2 - e^{+2r}(x_1 - x_2)^2 + (p_1 + p_2)^2]$$

$$W(x_1, p_1) = N\, exp[-\frac{2(x_1^2 + p_1^2)}{1 + 2\bar{n}}]$$ Reduced Wigner of thermal or max entropy state

Entanglement by Two mode squeezing

- Two mode squeezing operator produces entanglement between orthogonal quadratures of two modes.

$$\langle \hat{n} \rangle = \langle \hat{x}^2 \rangle + \langle \hat{p}^2 \rangle - 0.5 = |\alpha|^2 + sinh^2 r$$

$$H_{int} = i\bar{h}\kappa(\hat{a}_1^\dagger \hat{a}_2^\dagger e^{i\theta} - \hat{a}_1 \hat{a}_2 e^{-i\theta})$$ Interaction hamiltonian

$$\hat{U}(t,0) = exp(\gamma^* \bar{a}_1 \bar{a}_2 - \gamma \hat{a}_1^\dagger \hat{a}_2^\dagger)$$

$$\hat{a}_1(r) = a_1 coshr + \hat{a}_2^\dagger sinhr, \ \hat{a}_1(r) = a_2 coshr + \hat{a}_1^\dagger sinhr,$$

- So, we see that the input modes are entangled with finite squeezing parameter r

Bi-partite Entanglement of pure states

$\psi = \int_n C_n |n\rangle_A |n\rangle_B$: Scimdt form of composite state.

- Any state that cannot be written in schimdt form with only one coefficient(schimdt rank) is entangled or else separable.
- state with schimdt rank k =¿ schimdt rank of state of sub-system is k.

E.E=Tr$[\rho_A log(\rho_A)] = -\int_n C - n^2 log(C_n^2)$:

- Entanglement entropy(E.E)= Von Neumann Entropy of any subsystem, non. zero for inseparable system.
- E.E of a state s is asymptotic number of composite systems in state s required to produce max. entangled pair.

More ...

$\psi_{G.E} = \sqrt{1-\lambda} \int \lambda^{n/2} |n\rangle |n\rangle$: G.E = squeezed entangled state. in fock basis.

Mixed state Inseparability

Any state of composite system that can't be written as mixture of separable states is entangled or else inseparable.

Any state whose partial transpose(transpose w.r.t to one subsystem) is positive is ppt or else npt.

$npt \implies$ Entangled

violations of local realism $\implies$ entangled

(2,2) or (2,3) entangled $\leftrightarrow npt$

$(1 \times N)$mode gaussian $\leftrightarrow npt$

PPT criterion for bipartite Gaussian State

- As states are hermitian, taking transpose implies complex conjugating.
- Complex conjugating in Schrodinger equation implies time reversal.

$\mathrm{W}(x_1, p_1, x_2, p_2..x_N, p_N) \to w(x_1, -p_1...x_N, -p_N) \implies V^N \to \Gamma V^N \Gamma$

$\Gamma_a V^{N+M} \Gamma_a \not\geq \frac{i}{4}\Lambda$:NPT criterion for bipartite gaussian

- NPT criterion is sufficient not necessary as there exists PPT states which are entangled, the so called bound entangled states , i.e with Entanglement Entropy 0.

Generation of Entanglement

- Generation of entanglement is very similar to discrete case where computational states are passed through hadamard gate on first qubit and then subsequent C-Not Gates.

$$\hat{\mathcal{F}} |x\rangle_{pos.} = \frac{1}{\sqrt{\pi}} \int_{\inf}^{\inf} dy e^{2ixy} |y\rangle_{pos.} = |p = x\rangle_{mom} .,$$

- So, hadamard is going from x to p eigen states and CNot gate is beam splitter.

Working principle of Ent. Generator

We pass momentum eigen state through mode 1 and position eigen states through rest modes and then pass it through

$$B_{N-1,N} sin^{-1}(1/\sqrt{2}) B_{N-2,N-1} sin^{-1}(1/\sqrt{3}) .... B_{12} sin^{-1}(1/\sqrt{N}))$$

- Due to finite squeezing, the correlation s of resultant GHZ like states are $x_i - x_j \sim Ne^{-2r}$

Measurement of Entanglement

- As in discrete case, measurement of entaglement can be done by projection to Bell Basis, which can be done by inverting the circuit used to generate entanglement.
- We pass the modes through beam splitters as before of varying transmittance to balance power gain and then doing homodyne detection for quadrature measurement of p of first mode and x of rest modes.
- So, Generation and measurement of Entanglement just requires linear optics and squeezed states and done unconditionally as opposed to discrete case.
- The entanglement so generated can be verified through traditional EPR type approach or Phase space approach as mentioned in coming slides.

Verification Of Entanglement

- Einstein first busted Entanglement through the principle of local realism, i.e if one can predict the physical quantity without disturbing then that represents an objective reality and no action at distance is allowed.
- Since, systems far apart cannot influence each other and we can predict the outcome of a physical quantity of another system far apart from same measurement of one system if both are entangled , entanglement violates physical realism.
- John bell showed that physical realism imposed some constraints which in real experiments can be tested and those constraint came in form of Bell Inequalities.
- In traditional EPR approach measurement measurement of one quadrature is performed and from it the other is inferred and the variance is checked , if the variance is quite small it shows violation of Bell inequality.

Phase space approach

- Since, Wigner function which serve as hidden variable distribution of gaussian state of any mode is always positive, homodyne detection of quadratures can't reveal any entanglement,so non-gaussian measurements needs to be done.
- For their analysis using photon number parity measurements, the fact that the Wigner function is proportional to the quantum expectation value of a displaced parity operator of N mode.
- $W(\alpha) = (\frac{2}{\pi})^N \hat{\prod}(\alpha)\rangle$, $\hat{\prod}(\alpha) = \times_{i=1}^{N} \hat{D}_i(\alpha_i)(-1)^{\hat{n}_i}\hat{D}^\dagger(\alpha_i)$
- Thus wigner is a product of displaced parity operators corresponding to the measurement of an even parity +1 or an odd parity 1 number of photons in mode i. Each mode is then characterized by a dichotomic variablen.
- Now, similar to CHSH inequality we see that by taking the analogous ineuality $\hat{\prod}(0,0) + \hat{\prod}(0,\beta) + \hat{\prod}(\alpha,0) + \hat{\prod}(\alpha,\beta)$ we see max. violation for two mode inf. squeezed sate of 2.19

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Quantum Memory

- In order to store Quantum information we need to go from Electromagnetic Modes to atomic states.
- We store information in collective spin of atoms.
- In EM Wave Collective Stokes Variables serve as the spin counterpart.

$$\hat{\mathcal{S}}_{\mathcal{X}} = \frac{c}{2}\int_0^T d\tau[\hat{a}_+^\dagger(\tau)\hat{a}_-(\tau) + \hat{a}_-^\dagger(\tau)\hat{a}_+(\tau)],\ \hat{\mathcal{S}}_{\mathcal{Y}} = \frac{-ic}{2}\int_0^T d\tau[\hat{a}_+^\dagger(\tau)\hat{a}_-(\tau) - \hat{a}_-^\dagger(\tau)\hat{a}_+(\tau)],$$

$$\hat{\mathcal{S}}_{\mathcal{Z}} = \frac{c}{2}\int_0^T d\tau[\hat{a}_+^\dagger(\tau)\hat{a}_+(\tau) - \hat{a}_-^\dagger(\tau)\hat{a}_-(\tau)], \qquad [\hat{\mathcal{S}}_{|},\hat{\mathcal{S}}_{||}]=i\epsilon_{jkl}\hat{\mathcal{S}}_{\rangle} \text{ ETCR}$$

- For near maximum polarization of one variable we see that others behave canonically like position and momentum operators and hence by taking vaccum state as max. polarization state , spin-displaced and spin-squeezed sates can be formed analogously.

Continued...

- For Atomic states, collective spin operators of N atoms are therefore $\hat{F}_i = 1/N \sum_{i=1}^{N} \hat{F}_i^n$ with usual commuation relation of pauli operators.
- Then as before by taking max collective spin state we can define coherent state and squeezed states in limit N$\rightarrow \inf$
- A beam-splitter-like coupling between optical Stokes operators and the collective atomic spin can be achieved for strongly polarized off-resonant coupling.
- In particular, for light propagating along the z axis, the coupling is well described by the Jaynes-Cummings model. For a sufficiently off-resonant interaction, no population transfer will occur.
- Thus only second-order transitions can produce any effect, leading to an effective Hamiltonian. $H_{eff} \propto \hat{S}_z F_z$.
- This yeilds a quantum non-demolition probe of atomic collective spin, so we can store and read from memory efficiently.

Entanglement Distillation

- Entanglement is a resource like gold that can be enriched which, is producing M copies of more entanglement from N copies with less entanglement in asymptotic sense,i.e N,M $\rightarrow \inf$
- When the original states are mixed like after passing through noisy channel , the process is Entanglement Purification as in Swapping or its Entanglement conc. for pure states.
- Majorization a field in maths is used as deterministic way to concentrate entanglement.
- Schimdt projection method is a probabilistic method where measurement projects to subspace with coomon schimdt coefficient.
- In C.V Schimdt decomposition like photon number non demolition measurement projects to space of higher entanglement.
- Gaussian States cannot be enriched with just gaussian operations.

Quantum Teleportation

- Transfer of Quantum state from A to B beyond possible through classical channel.
- Classical channel used as subroutine , so doesn’t violate Special relativity.
- No inf. is gained in perfect teleportation of input state.

Working Principle

Analogous to Teleportation of qubits, entanglement is established between A and B. A then passes the input mode and here entangled mode through Beam splitter and does bell detection on output i.e homodyne detection of x quadrature of one and p of other to produce an random photocurrent with gain g.

- Quantum Teleportation is an useful subroutine in Entanglement swapping for reliable Quantum communication.

Equations of Teleportation

$$\hat{x}_\mu = \frac{1}{\sqrt{2}}(\hat{x}_{in} - \hat{x}_1)\ ,\ \hat{p}_\mu = \frac{1}{\sqrt{2}}(\hat{p}_{in} - \hat{p}_1)$$

$$\hat{x}_\upsilon = \frac{1}{\sqrt{2}}(\hat{x}_{in} + \hat{x}_1)\ ,\ \hat{p}_\upsilon = \frac{1}{\sqrt{2}}(\hat{p}_{in} + \hat{p}_1)$$

$$\hat{x}_2 = \hat{x}_{in} - (\hat{x}_1 - \hat{x}_2) - \sqrt{2}\hat{x}_\mu = \hat{x}_{in} - \sqrt{2}e^{-r}x_2^0 - \sqrt{2}\hat{x}_\mu$$

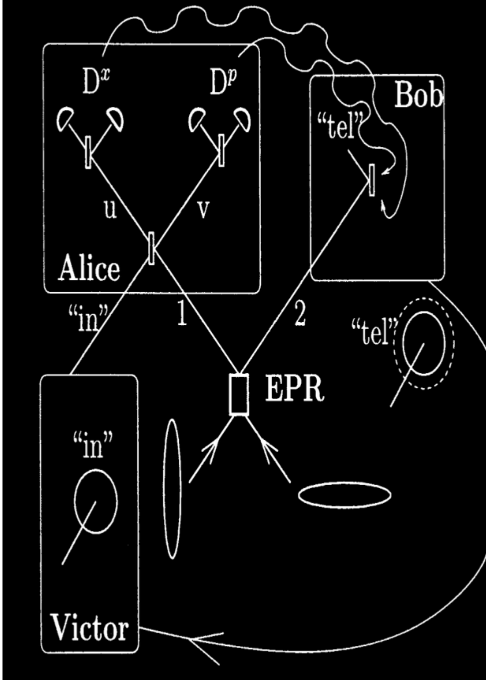
$$\hat{p}_2 = \hat{p}_{in} - (\hat{p}_1 - \hat{p}_2) - \sqrt{2}\hat{p}_\mu = \hat{p}_{in} - \sqrt{2}e^{-r}p_2^0 - \sqrt{2}\hat{p}_\mu$$

$$\hat{x}_2 \rightarrow \hat{x}_{tel} = \hat{x}_2 + g\sqrt{2}\hat{x}_\mu$$

$$\hat{p}_2 \rightarrow \hat{p}_{tel} = \hat{p}_2 + g\sqrt{2}\hat{p}_\mu$$

$$\hat{x}_{tel} = g\hat{x}_{in} - \frac{g-1}{\sqrt{2}}e^{r}\hat{x}_1^0 - \frac{g+1}{\sqrt{2}}e^{-r}\hat{x}_2^0$$

$$\hat{p}_{tel} = g\hat{p}_{in} - \frac{g-1}{\sqrt{2}}e^{r}\hat{p}_1^0 - \frac{g+1}{\sqrt{2}}e^{-r}\hat{p}_2^0$$



Verification Of Teleportation

- Victor a third party analyzes the efficiency of teleportation through fidelity of output with input state.

$F_{av} = \int P(|\phi_{in}\rangle) \langle\phi_{in}| \hat{\rho}_{tel} |\phi_{in}\rangle \, d\phi_{in}$: P is probability dist. of input alphabets

- F_{av} of input alphabets of coherent states with uniform distribution is Q function of teleported state which is wigner function of state convoluted with unit gaussian.

$$F=\pi Q_{tel}(\alpha_{in}) = \frac{1}{2\sqrt{\sigma_x \sigma_p}} exp[-(1-g)^2(\frac{x_{in}^2}{2\sigma_x} + \frac{p_{in}^2}{2\sigma_p})]$$

$$\sigma_x = \sigma_p = \frac{1}{4}(1+g^2) + \frac{e^{+2r}}{8}(g-1)^2 + \frac{e^{-2r}}{8}(g+1)^2$$

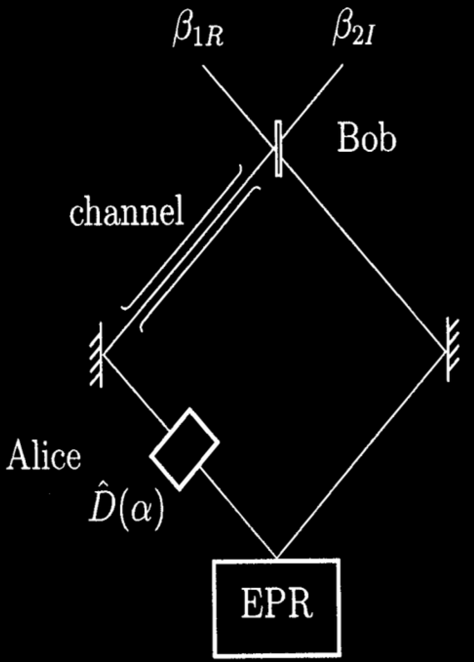
- W_{tel} is convolution of W_{in} with gausisian of variance e^{-2r}.
- So with only classical channel, there is a noise of 1 variance which comes from 1: Measurement of $p_i n$ and $x_i n$ simultaneously, 2: Creation of coherent state from classical signal.

Dense Coding

- Opposite of teleportation, used to double the classical channel capacity by sending a Quantum state.

Working Principle

Alice and Bob share entanglement which is two mode squeezed state. Alice modulates her mode with classical signal(displaces here mode by α), and then sends her mode to Bob. Bob does a Bell detection. It is similar to Discrete variable Dense coding as displacement done by Alice is changing x$_a - x_b = 0 correlation to x_a - x_b = \alpha$ correlation detected through bell measurement.



Dense Coding Equations

$I^{dense}(A:B) = \int d^2\beta d^2\alpha p_{\beta|\alpha} p_\alpha ln \frac{p_{\beta|\alpha}}{p_\beta} = ln(1+\sigma^2 e^{2r})$: Mutual Information

$p_{\beta|\alpha} = \frac{2e^{2r}}{\pi} exp(-2e^{2r}|\beta - \alpha/\sqrt(2)|^2)$: For ideal homodyne detection

$p_\alpha = \frac{1}{\pi\sigma^2} exp(-|\alpha|^2/\sigma^2)$ n = $\sigma^2 + sinh^2 r$

$p_\beta = \frac{2}{\pi(\sigma^2+e^{-2r})} exp(\frac{-2|\beta|^2}{\sigma^2+e^{-2r}})$

$C^{dense} = ln(1+\bar{n}+\bar{n}^2) = 4r$, for large squeezing by optimizing with $\bar{n} = e^r sinhr$

Classical Channel capacity by putting $\bar{n} = e^r sinhr$ in eq. x is 2r.

- Contrast to teleportation, where for any $r > 0$, it beats classical counterpart, in dense coding for r<0.7809, it is worse than classical coding.

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Quantum Cloning

- Quantum no-cloning theorem which, states that no arbitrary state can be replicated precisely, is used for modelling eavesdropper in Quantum Cryptography.
- In N-M Quantum Cloning we try to produce(M>N) copies from N copies of original state and M copies have high fidelity with original state.
- Universal cloning means optimally copying arbitrary quantum states with the same fidelity independent of the particular input state.
- Werner by axiomatic approach proved that the Fidelity bound for arbitrary dimension is $\frac{N(d-1)+M(N+1)}{M(N+d)}$ for a universal cloning device of dimension d.
- Asymmetric Quantum Cloning means the Quantum information is asymmetrically distributed among resultant modes where some have less than universal fidelity bound and some have more.

$$\hat{\rho}_a = (1-A^2)\left|s\right\rangle_{aa}\left\langle s\right| + \frac{A^2}{d}\mathbb{I}_d, \hat{\rho}_b = (1-B^2)\left|s\right\rangle_b\left\langle s\right| + \frac{B^2}{d}\mathbb{I}_d \text{ , } A^2+B^2+2AB/d = 1$$

Working Principle of Quantum Cloning

- In discrete case, 1-2 cloning is performed using four C-Not gates acting pairwise on a:input state, b and c:entangled states.
- in continpus case, the entangled states become two mode squeezed states and the CNot gates become translation operators($|x\rangle\,|y\rangle \rightarrow |x\rangle\,|x+y\rangle$)
- Hence the Cloning process can be summed by $U_{abc} = exp[-2i(\hat{x}_c - \hat{x}_b)\hat{p}_a]exp[-2i\hat{x}_a(\hat{p}_b + \hat{p}_c)]$.
- In continuous the information preserve formula becomes $A^2 + B^2 + 4AB/\sqrt{4 + 2sinh^2 2r} = 1$
- In infinite squeezing and symmetric cloning, the fidelity bound becomes N/M which is putting d to $\inf$ in werner formula.
- This result is essentially classical and can be simulated classically as by inputting the original state and a random state($\mathbb{I}$) it outputs original state and random state through b or c probabilstcally(coin toss) and absence of entanglement between output states of C.V cloner also confirms this.

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Fidelity bound for Gaussian States

- Restricted to Coherent states, the fidelity bound improves $F_{coherent} = \frac{MN}{MN+M-N}$
- it is proved through a method analogous to bloch vector shrinking method employed by Preskill i.e $\hat{\rho}_a = \mathbb{I} + \vec{S}.\vec{\sigma} \rightarrow \mathbb{I} + \eta(N,M)\vec{S}.\vec{\sigma}$ where η is the shrinking factor.

Assumptions

In concatenation of Cloning devices, the shrink factor multiplies and $\eta(N,\inf)$ equals quantum estimate through measurement(η_{meas}). Intuitively the second assumption means that the amount of information we can gain of a state from N copies of it equals the shrink factor for going from N copies to infinity.

$\eta(N,M)\eta(M,L) \leq \eta(N,L)$ and $\eta(N,\inf) = \eta(N)_{mean}$

$$\eta(N,M) \leq \frac{\eta(N,\inf)}{\eta(M,\inf)} = \frac{\eta_{meas}(N)}{\eta_{meas}(M)} \rightarrow F = 1/2 + \eta(N,M)/2, \eta(N,M) = \frac{N(M+2)}{M(N+2)}$$

Continuous Variable Generalization of Shrinking Method

- So, as bloch vector reduces in length due to noise, its C.V counterpart is then that the noise in concateation sums up. i.e $\lambda_{clon}(N, M) \geq \lambda_{clon}(N, \inf) - \lambda_{clon}(M, \inf)$
- λ can be inferred from Quantum Estimation theory because it equals the quadrature variance of an optimal joint measurement of $\hat{x}$ and $\hat{p}$ on N identically prepared systems.
- For N modes, λ_{meas} equals N/2 as for one mode noise comes from one unit of vaccum and one unit due to simulataneous measurement of quadratures.

$$\lambda_{clon}(N, \inf) = \lambda_{meas}(N) \implies \lambda_{clon}(N, M) = \frac{M-N}{2NM}$$

$$F_{coherent} = \frac{MN}{MN+M-N} \qquad (\sigma_x = \sigma_y = \lambda_{clon}(N, M))$$

- For Squeezed states the noise also get squeezed in proportion by concatenation and as this reqquires knowledge of squeezing parameter , this device is not Universal.

Quantum Cryptography and Quantum Secret Sharing

- Quantum key distribution is creating same bit string or key on sender and receiver unconditionally protected from any eavesdropper by laws of Quantum Mechanics.
- Done through Preparation and Measurement scheme and Entanglement scheme.
- In entanglement scheme, there is state preparation at distance by sharing entangled pair and measuring in randomly chosen basis.
- In preparation and measurement scheme, indistinguishablity of non-orthogonal states is exploited to do cryptography.
- For creation of such high correlation between sender and receiver can only be achieved through Quantum correlation or entanglement.
- So, in any Entanglement scheme(even in prepare and measure) scheme first share quantum resource and then check for sufficient entanglement.

Contd......

- For, Continuous Variable Entanglement witness is given by Duan Criterion.
- Ralph, tried entanglement scheme where bit string was encoded in intense squeezed light and then entangled. Protected from non collective attacks(There's in total individual , collective and coherent attacks)
- Extension of Quantum Key Distribution. Key is established among many parties and protected from eavesdropper.
- GHZ state stores Quantum Information among three parties.
- K-n Quantum Secret sharing established among n parties s.t any k of them can only unlock.In CV GHZ

Universal Quantum Computation

- Search for smallest set of simple unitary operators that can be used to build any arbitrary unitary acting on Continuous Variables.
- Systems with C.V have infinite-dimensional Operator space.
- So, we try to create arbitrary polynomial hamiltonian.(polynomial decomposition).
- Just x or p produce linear shift in p or x quadrature respectively.

$e^{-iA\delta t}e^{-iB\delta t}e^{iA\delta t}eiB\delta t = e^{-i[A,B]\delta t} + \mathcal{O}(\delta^3)$: Hausdorff Campbell Formula

- Provided I can apply $\pm H_i$, I can produce $\pm i[\mathrm{H}_i, H_j]$,$\pm i[\mathrm{H}_K,[H_i, H_j]]$ etc.
- with $\pm x$, $\pm p$ we can produce ax+bp+c hamiltonians.
- $\mathcal{H}$ = $x^2 + p^2$ produces linear periodic mixing of quadratures by phase shifters.

$\hat{x} \rightarrow \hat{x}cos(t) - \hat{p}sin(t)$, $\hat{p} \rightarrow \hat{p}cos(t)\hat{x} + sin(t)$: $\mathcal{H} \rightarrow -\mathcal{H}$ if applied for time $4\pi - \delta t$

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Continued...

- $\hat{S} = \hat{x}\hat{p} + \hat{p}\hat{x}$ or squuezing stretches the quadratures orthogonally.
- Kerr Hamiltonian $\mathcal{H}^2$=$(\hat{x}^2 + \hat{p}^2)^2$ is a third order nonlinear process and its commutation with existing optical elements shows that we can produce any third order hamiltonian.
- Inductively one can now show that with just squeezers, kerr non linearity, phase and quadrature shifters any polynomial hamiltonian can be constructed.
- Now, we look for multi-mode Hamiltonian more specificallly two mode.
- $\hat{B}_{ij} = \hat{p}_i\hat{x}_k - \hat{x}_i\hat{p}_j$ produces linear periodic mixing of quadratures of different mode.
- So, in contrast to qubits, with just linear optics and kerr non-linearity on single mode and linear mixing of two modes, any hamiltonian can be constructed.
- Analogous to Qubit, it is Qunat for CV quantum information given by Von-Neumannn entropy.

Gottesman-Knill Theorem for C.V

- But some Quantum Computations can be simulated efficiently on classical computer.
- In qubit the theorem states that starting with computational basis , and restricting gates to like Pauli, hadamard and CNot and projective measurement in computational basis can be done efficiently on classical.
- In C.V case, we use position eigen states as computational basis, position translation and momentum kicks as X and Z pauli operators respectively.
- As Hadamard gate is Discrete fourier transform , we use $\hat{F}=e^{i\pi(x^2+p^2)^2/2}$ as our hadamrd gate and $e^{i\eta\hat{p}^2}$ as phase gate.
- Now, analogous to Discrete case, we find operators that stabilize the subspace we are concerned with i.e the subpspace is eigenspace of value 1 and then look for evolution of the generators of these stabilizers i.e in Heisengberg picture rather than keeping track of exponential overheads of state evolution.
- Using tensor product of npauli operators as generators, we see that its evolution goes to tensor product of pauli operators, so we keep track of 2n real coefficients(x,p).
- Projective measurement in position eigen states is done by simulating the

End

THE END