SUMMER-2019 PROJECT REPORT

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1. THE VICSEK MODEL

1.1 Introduction

The Vicsek model [1] is a continuous space, discrete time model of self-propelled particles which act according to a universal and simple rule to determine their behaviour: each particle moves with a constant absolute velocity, but at each time step its orientation is updated to the average of the particles in its interaction radius, r with an additional random contribution. We will be concerned with particle motion in two dimensions, however the concept is easily extendible to n-dimensions.

A direct analogy can be drawn with ferromagnetic type models (for example, the XY model since we are looking at 2 dimensions), with the rule for the tendency of neighbouring sites to align spin corresponding to the tendency of particles in the Vicsek model to align their neighbours direction of motion. One key difference between these models is that particles in the Vicsek model are not constrained to lattice sites, as such the model is inherently dynamic, allowing local density fluctuations. The inclusion of a noise is analogous to temperature in ferromagnetic type models. The Vicsek model is interesting particularly for application to biological systems as a minimal model for social clustering, for example with birds or fish, who form flocks and schools and display a natural collective behaviour, and even bacteria who may act cooperatively to survive. It may surprising then that a simple model based on a single local rule can produce interesting global dynamics, as the Vicsek model does.

2.2 Simulation

Simulations were performed on a two dimensional square cell of size, D × D with periodic boundary conditions. Certain fixed parameters were chosen from the original Vicsek paper, to facilitate comparison. The global density, ρ was kept 16.0. This inferred values for cell size, D for varied number of particles, N via $\rho = N/L^2$. The interaction radius, r was fixed at 1.0 and the constant speed, v_0 at 0.03. Since this model drives the particles with a constant speed, the net momentum of interacting particles is not conserved. The noise parameter was varied in the range [0.0, 5.0] to gain good resolution of the variation in order parameters. We need an order parameter to predict how the system of particles orients itself as it evolves. If the average velocity of a random particle flow is considered, it is intuitive that is there is no preferred direction and all particles are moving randomly, then individual contributions will cancel to leave close to zero, however if there is a well defined order and direction of motion, this will be close to unitary (if normalised). Therefore the average normalised velocity is defined as the order parameter:

$$v_{abs} = (1/Nv_0)\ \left|\sum_{i=1}^{n} \mathrm{v}\right|$$

To start the simulation, the initial conditions are defined. A random initial configuration of particle positions x,y and orientations θ were used. At each time step the positions of each particle based on its velocity and the orientation of each particle based on the mean orientation of its neighbours within its interaction range were updated. The orientation was then augmented by a random number distributed with uniform probability on the interval $[-\eta/2, \eta/2]$, where η is the magnitude of the noise as was selected for the run. The new velocity values for the particles were defined by this way. The update of these values were performed simultaneously each time step. The order parameter was also calculated at each time step, based on the updated particle values(the particle plot and the order parameter plot were kept as separate simulations due to device restrictions). The Python code is attached to this report, which should provide some idea as to how this model was simulated specifically.

2.3 Typical Configurations

When the velocity field of the particles are plotted on a two dimensional interaction area, a few macroscopic configurations which are qualitatively different from one another are generated. This is an example of global behaviour variation due to differences in the local interaction rule.

Figure 1(a-d) show a selection of such snapshots in order to see the phase of the system. The velocity of each particle is represented by an arrow representing the orientation.

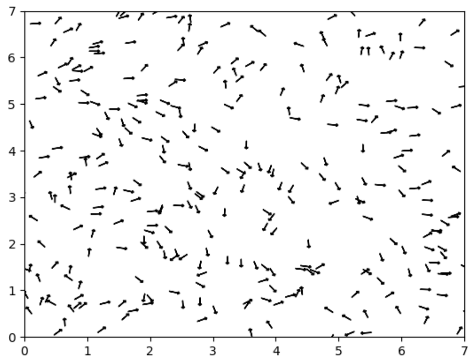
(a) At t = 0 there is a random distribution of positions and velocities for the particles. This corresponds to the high noise and low density.

(b) For low noise and low density, particles slowly form clusters who all locally move in the same direction, however the global distribution of cluster orientations is random.

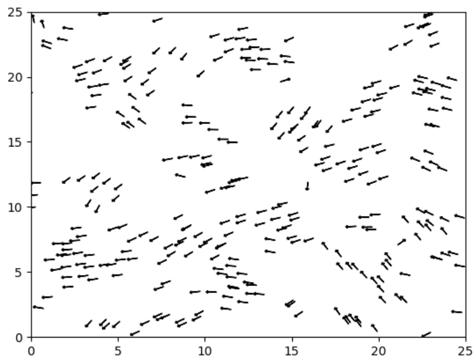
(c) For high density and high noise, there is some correlation between the particles, due to the sheer proximity of the particles, however it is largely drowned out by random fluctuations.

(d) For large density and low noise, the motion becomes macroscopically ordered, with particles all moving in the same direction.

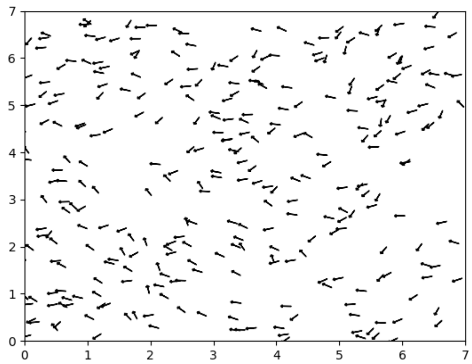
Since the initial condition is always a random distribution of positions and velocities, this symmetry is spontaneously broken as the particles select a direction randomly.



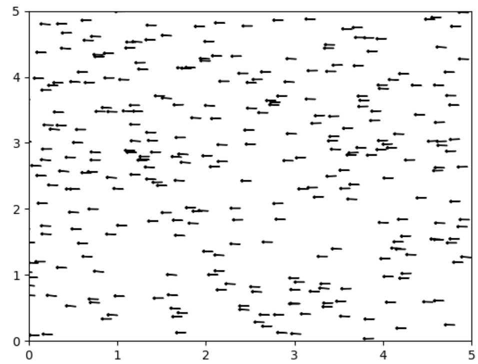
(a)



(b)



(c)



(d)

Figure-2: Configurational snapshots of velocity field. For all, N = 300

(a) t = 0, η = 2.0, D = 7.0 (b)D = 25, η = 0.1 (c) D = 7, η = 2.0 (d) D = 5, η = 0.1

2.4 Phase Transition

Figure 2(a) shows the variation of the order parameter, average velocity v_{abs}, as noise η is varied between [0, 5.0] in steps of 1.0. It is evident that the system goes from a state of high order when η is low, that is that there is a defined global direction, to a state of low order, with no defined global direction.

Also, Figure-2(b) shows the variation of the order parameter, average velocity, v_{abs} as the density, ρ is varied between [0.1, 4.0].

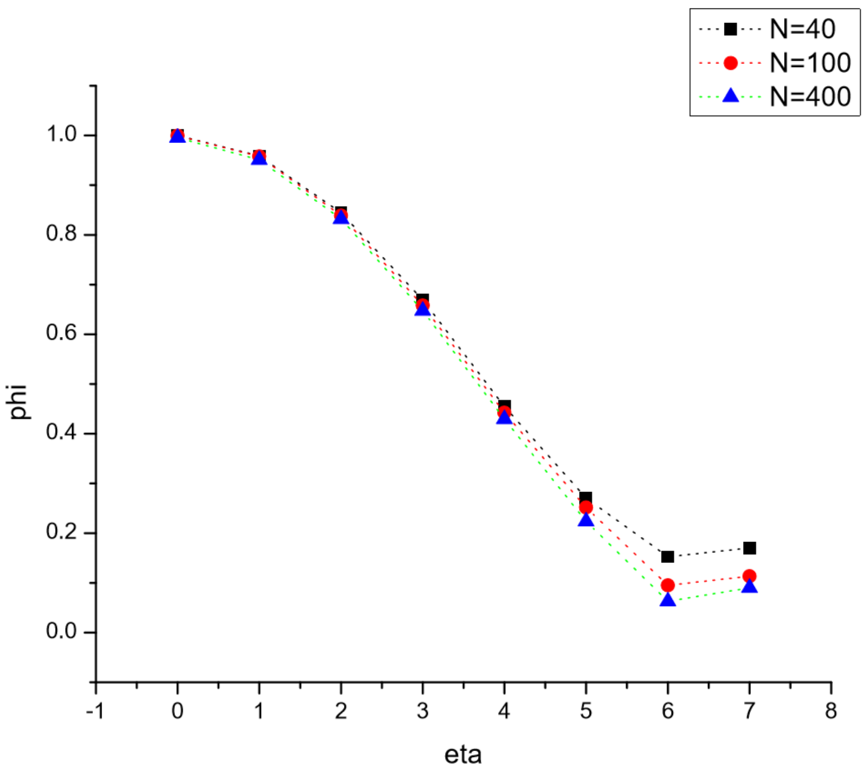


Figure-2(a): The order parameter, average velocity, v_{abs} versus the noise, η for different values of particle numbers, N and fixed density, ρ = 16.0 (varied cell size, D)

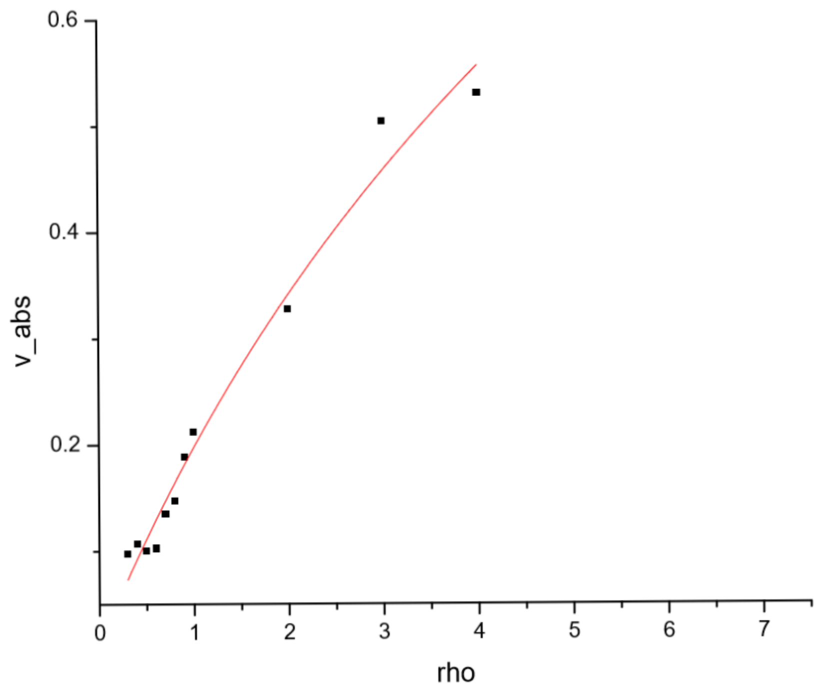


Figure-2(b): The order parameter, average velocity, v_{abs} versus the density, ρ and constant noise, η = 3.0

2.5 References

[1] Vicsek, et al.,”Novel Type of Phase Transition in a System of Self Driven Particles”, Phys. Rev. Lett. 75:1226, 1995.