PH3105 Problem Set 4

Q 1) In the class we saw that when the number of points is large, the error in the composite trapezoidal formula is given by

$$E_n \approx -\frac{h^2}{12}\left[f'(b) - f'(a)\right]$$

This leads to the so-called "Corrected trapezoidal formula"

$$\int_a^b f(x)\,dx = h\left[\frac{f(a)+f(b)}{2} + f_1 + f_2 + \ldots + f_{n-1}\right] - \frac{h^2}{12}\left[f'(b) - f'(a)\right]$$

Write a program that will utilize this formula to estimate the integral

$$\int_0^\pi e^x \cos x\,dx$$

and determine the actual error for $n = 2, 4, 8, \ldots, 128$. By plotting an appropriate graph find the dependence of this error on h.

Q 2) The trapezoidal formula is based on interpolation of the integrand with a linear polynomial (a straight line). Derive the formula that will involve interpolation with a cubic polynomial (take four equally spaced points as the nodes, a and b both inclusive). Write a program that implements this integration algorithm. Use it to calculate the integral

$$\int_0^h e^x dx$$

for $h = 1, \frac{1}{2}, \frac{1}{4}, \ldots, \frac{1}{128}$. From your results deduce the degree of the error in terms of h.

Q 3) Write a program that will implement n-point Hermite interpolation. Use it to interpolate the Runge function (problem set 1) for $n = 10$ and compare your interpolation with the 20 point Lagrange interpolation.