PH3105 Problem Set 5

Q 1) Use 2-point, 3-point and 4-point Gauss quadrature to estimate $\mathrm{erf}\,(1.5)$ where the function $\mathrm{erf}\,(a)$ is defined by

$$\mathrm{erf}\,(a) \equiv \frac{2}{\sqrt{\pi}} \int_0^a e^{-x^2}\, dx$$

Note : the exact value of $\mathrm{erf}\,(1.5)$ is 0.966105.

Q 2) An improper integral where one or both the limits diverge to $\pm\infty$ can be evaluated numerically by an appropriate substitution of variables to render the range of integration finite. Do this (using 4-point Gauss quadrature on the final proper integral to estimate

$$\int_2^\infty \frac{dx}{x\,(x+2)} \qquad \text{and} \qquad \int_0^\infty e^{-y} \sin^2 y\, dy$$

Q 3) Use Richardson extrapolation to estimate the integral

$$\int_0^\pi \sin^2 x\, dx$$

and verify that the error is $\mathcal{O}\left(h^4\right)$.

Q 4) Study Romberg integration (Wikipedia may be a good place to start) and write a program to implement it.