PH3105 Problem Set 6

Q 1) Solve the following problem over the interval from $x = 0$ to 1 using a step size of 0.1 where $y(0) = 1$. Display all your results on the same graph.

$$\frac{dy}{dt} = (1 + 4t)\sqrt{y}$$

(a) Analytically. (b) Euler's method. (c) Heun's method.(d) Ralston's method. (e) Fourth-order RK method.

Q 2) Solve the following problem numerically from $t = 0$ to 3:

$$\frac{dy}{dt} = -2y + t^2, \qquad y(0) = 1$$

For this use a step size $h = 0.1$ and use a third order RK algorithm

$$y_{n+1} \approx y_n + \frac{h}{6}\left(p_n + 4q_n + r_n\right)$$

where

$$\begin{aligned} p_n &= f\left(y_n, t_n\right) \\ q_n &= f\left(y_n + \frac{h}{2}p_n, t_n + \frac{h}{2}\right) \\ r_n &= f\left(y_n - p_1 h + 2p_2 h, t + h\right) \end{aligned}$$

Plot both your solutions and the error in separate graphs.

Q 3) Use RK4 to solve the system of equations

$$\begin{aligned} \frac{dy}{dt} &= -2y + 5e^{-t} \\ \frac{dz}{dt} &= -\frac{yz^2}{2} \end{aligned}$$

over the range $t = 0$ to 0.4 using a step size of 0.1 with $y(0) = 2$ and $z(0) = 4$.

Q 4) Consider the ODE

$$\frac{dy}{dx} + 0.6y = 10\exp\left(-\frac{(x-2)^2}{2\,(0.075)^2}\right)$$

with the initial condition $y(0) = 0.5$.

Solve this equation numerically using RK4 from $x = 0$ to $x = 4$, using a step size $h = 0.1$. Repeat with $h = 0.05$ and use this to estimate the error in your solution as a function of x.

Q 5) Use RK4 with adaptive step size to solve the above equation using an initial step size of 0.5 and an error bound $\epsilon = 0.00005$.

Q 6) An alternative to halving the step size to estimate the error is to use two different order RK methods to estimate y_{n+1}. The problem is that this tends to increase the number of function evaluations. However, the Runge-Kutta-Fehlberg algorithm cleverly circumvents this by reusing the slopes evaluated for RK4 to calculate an RK5 estimate.

One variant of this is the Cash-Karp method. Read up about the method on the net (the wikipedia article on this topic is sufficient for our current purpose) and write a program that solves the equation in question 4.