PH3105 Problem Set 7

Q 1) Solve

$$\frac{dy}{dt} = -15y, \qquad y(0) = 1$$

using the explicit Euler method for the same step sizes that were discussed in the class for the implicit method. Compare your solutions with the analytic solution by plotting them together.

Q 2) a)Solve

$$\frac{dy}{dt} = -1000y + 3000 - 2000e^{-t}, \qquad y(0) = 0$$

analytically.

b) Now solve this numerically using the explicit Euler method. Use step sizes of $h = 0.0005$ and $h = 0.0015$ to solve for y between $t = 0$ and $t = 0.006$. Plot your solutions alongside the exact one. Comment on the stability of the solution.

c) Now repeat this using the implicit Euler method with a step size of 0.05 between $t = 0.05$ and 0.4

Q 3) Solve the following initial-value problem from $t = 1.5$ $t = 2.5$

$$\frac{dy}{dt} = -\frac{2y}{1+t}$$

Use the fourth-order Adams predictor-corrector method. Employ a step size of 0.5 and the fourth-order RK method to predict the starting values, given $y(0) = 2$.

Q 4) Solve the heat transfer equation

$$\frac{d^2T}{dx^2} + h'(T_a - T) = 0$$

where $T(0) = T_1$ and $T(L) = T_2$ analytically. Use the values

$$L = 10,\ T_a = 20,\ T_1 = 40,\ T_2 = 200 \text{ and } h' = 0.01$$

Use the shooting method (along with RK4) to solve this numerically, with a step size of 0.1.

Q 5)

Solve

$$\frac{d^2T}{dx^2} + h''(T_a - T)^4 = 0$$

where $h'' = 5 \times 10^{-8}$ and $T_a = 20$ subject to the boundary conditions $T(0) = 40$ and $T(10) = 200$.