PH3202 Problem Set 2

Q 1) Find the surface charge density induced by a point charge q placed at a distance d from an infinite grounded conducting plate. *(Note that the surface charge density of a conductor is given by $\epsilon_0 E_n$ - where E_n is the component of the electric field normal to the conductor.)*

By direct integration show that the net induced charge is $-q$.

Q 2) A point charge $+q$ is released from rest from a point at a distance d from an infinite grounded conducting plane. Determine the time in which the charge will hit the plane. Assume that the charge is moving slowly enough (and the corresponding redistribution of the induced surface charge distribution is slow enough) for us to treat this problem using electrostatics.

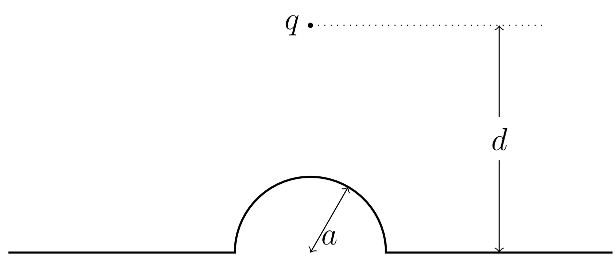
Q 3) Find the surface charge density induced on a conducting sphere of radius a by a charge q at a distance d, $(d > a)$, from the center of a sphere.

Integrate this charge density to find the net surface charge induced. Also give an argument based on Gauss law why this matches the image charge.

By a direct integration find the net force exerted by the grounded sphere on the point charge q.

Q 4) Consider a charge $+q$ placed *inside* a grounded conducting sphere of radius a at a distance d, $(d < a)$, from the center of a sphere. Find the potential everywhere inside the sphere by using the method of images. Hence determine the surface charge density induced on the sphere. Calculate the net charge induced by direct integration. Can you explain why this result does not match the size of the image charge?

Q 5) An infinite planar conducting sheet has a hemispherical bulge of radius a. The plate is grounded and a charge $+q$ is located at a distance d directly above the center of the hemisphere as shown in the figure.



Construct and appropriate image charge system and hence determine the potential at all points above the sheet.

Q 6) [The second uniqueness theorem] Consider a system that consists of a bunch of conductors (with surfaces Σ_1, Σ_2 , ...). The net charges Q_1, Q_2, ... on the conductors are specified (note - only the net charges are specified and not how they are distributed), and so is the charge distribution $\rho\left(\vec{r}\right)$ in the region τ outside the conductors. The potential on the outer surface of τ is also specified.

Let V and V' both be potentials consistent with this situation. Argue that this means that $u = V - V'$

- obeys the Laplace equation,
- takes constant values on each Σ_i(not necessarily the same constant)
- and satisfies
$$\oint_{\Sigma_i} \frac{\partial u}{\partial n} dS = 0$$
on each Σ_i (think of the connection between surface charge density on a conductor and the electric field at its surface).

Use this to prove that the potential in such a system is unique.