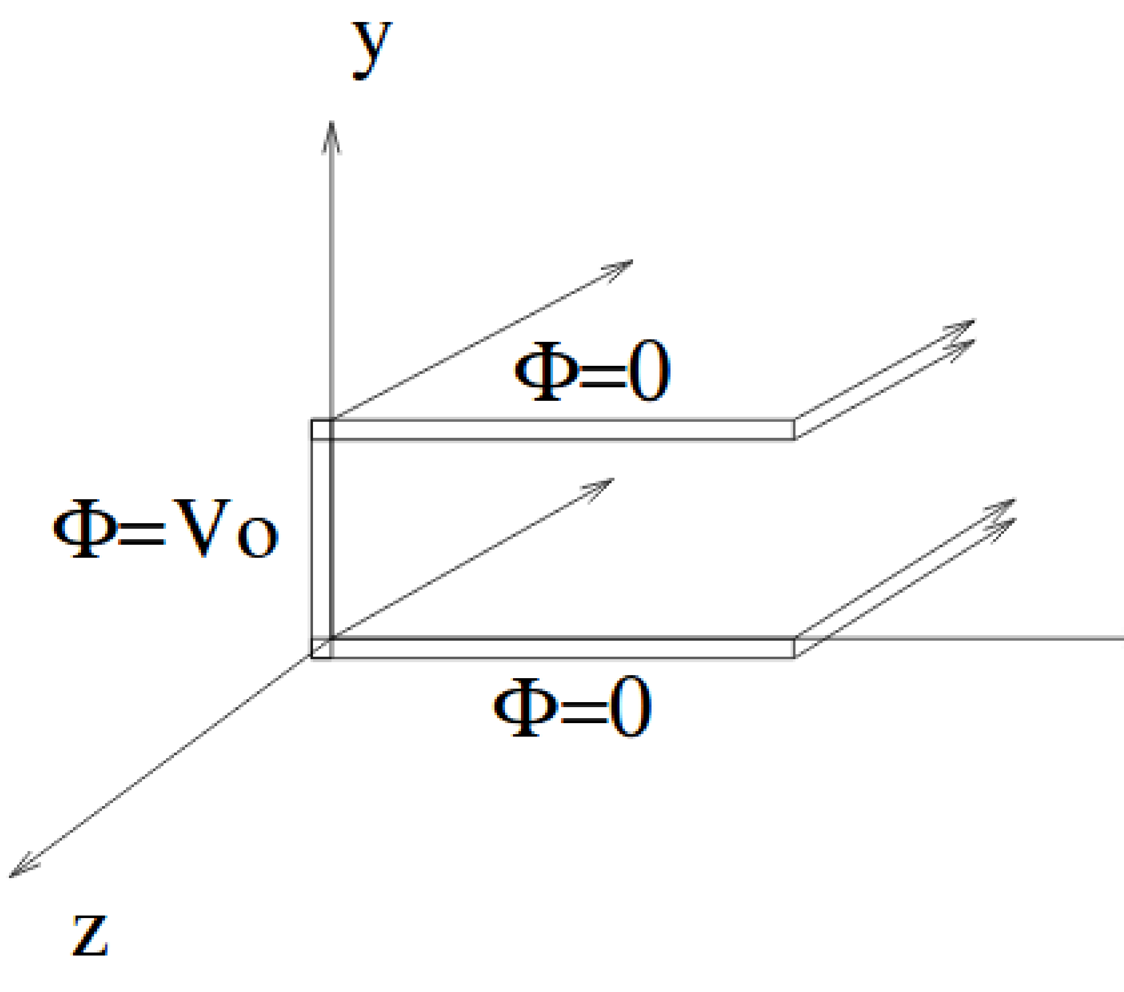
PH3202 Problem Set 3

Q 1) A slot is created by two infinite metal plates lying parallel to the XZ plane, one at $y = 0$, and the other at $y = L$

with the left end at $x = 0$ closed off with an infinite strip insulated from the two plates, and maintained at potential $V_0(y)$ as shown in the figure below:



Find the potential everywhere in the slot ($x \geq 0$, $0 \leq u \leq L$).

Note : The potential in this problem depends only on x and y due to the translation symmetry in the X direction - so this involves the 2-D Laplace equation.

Q 2) The next question again concerns the Laplace equation in 2 dimensions, but in a slightly esoteric coordinate system. The elliptic coordinate system is described by the coordinates (σ, τ) which are related to the Cartesian coordinates (x, y) by the relation

$$x = c \cosh \sigma \cos \tau, \;\; y = c \sinh \sigma \sin \tau$$

where c is a constant.

(i) Show that lines of constant σ are ellipses, while those of constant τ are hyperbola with the same foci. Hence suggest physical problems where these coordinates could be useful.

(ii) Prove that the 2-D Laplacian operator in this coordinate system is given by

$$\nabla^2 = \frac{1}{c^2 \left(\sinh^2 \sigma + \sin^2 \tau\right)} \left(\frac{\partial^2}{\partial \sigma^2} + \frac{\partial^2}{\partial \tau^2}\right)$$

(iii) Solve the equation $\nabla^2 u = 0$ in the (σ, τ) coordinate system and write down the most general solution. *Remember that τ and $\tau + 2\pi$ refer to the same point!*

Q 3) Find the Dirichlet Green function for the Poisson equation in upper half plane $z > 0$. *Hint : remember the point charge in front of a grounded* plate.

Use this Green's function to solve the following problem : "A ring of radius a is insulated from the rest of the infinite grounded plate and is maintained at a constant potential V_0. Find the potential everywhere on the axis of the ring."

Remember : the normal derivative of G_D that you will need for this problem *is related to the surface charge density in the corresponding image problem.*

Q 4) Prove that the Dirichlet Green's function $G_D(\vec{r}, \vec{r}')$ is symmetric.

Q 5) Write a program to calculate the value of the potential which satisfies the Laplace equation inside the cube of side L where one face is held at a constant potential V_0 and the other five faces are grounded. Use this to determine (accurate to at least 8 decimal places) the value of the potential at the center of the cube.