PH3202 Problem Set 5

Q 1)a) Show that when $|\vec{r'}| \ll |\vec{r}|$, we have, up to the quadratic power of $\frac{r'}{r}$,

$$\frac{1}{|\vec{r}-\vec{r'}|} = \frac{1}{r} + \frac{\vec{r'}\cdot\hat{r}}{r^2} + \frac{1}{2r^3}\left[3\left(\vec{r'}\cdot\hat{r}\right)^2 - r'^2\right]$$

b) Use this to derive the qudrupolar approximation to the potential at $\vec{r}$ due to a system of charges q_1, $q_2, \ldots, q_i, \ldots$ positioned at $\vec{d_1}, \vec{d_2}, \ldots \vec{d_i}, \ldots$ where $|\vec{r}| \gg \left|\vec{d_i}\right|$. This gives us the Cartesian version of the multipole expansion (up to the quadupole term).

c) Write down the form that this takes for a continuouse source charge.

Q 2) Find the dipole moment of the system of four point charges : q at $(a,0,0)$, q at $(0,a,0)$, $-q$ at $(-a,0,0)$ and $-q$ at $(0,-a,0)$. Hence find the leading approximation to the potential at a point (x,y,z) where $x^2+y^2+z^2 \gg a^2$.

Q 3) Two wires of equal length l carry linear charge densities of $+\lambda$ and $-\lambda$ respectively. They are joined together at their midpoints and make an angle θ with each other. Choose their two angle bisectors as the X and Y axis respectively and determine the Cartesian quadupole tensor for this system. Hence find the potential at a distance $r \gg l$ from the center.

Q 4) Calculate all the spherical multipole momentsq_{lm} of a sphere of radius a which carries a volume charge density given by

$$\rho\left(r,\theta,\phi\right) = \rho_0\left(1-\frac{r}{a}\right)\cos^2\theta\sin\theta\cos 2\phi$$

and hence find the potential at a point outside the sphere.

Q 5) a) Find the Cartesian quadrupole moment tensor for a system of three charges : $+2q$ at the origin, and $-q$ each at $(0,0,\pm a)$.

b) If the coordinate system is rotated about the Y axis through an angle of 45°,the new positions for the two $-q$ charges will be $\pm\left(-\frac{a}{\sqrt{2}},\,0,\,\frac{a}{\sqrt{2}}\right)$ respectively. Find the quadrupole moment tensor in these coordinates.

c) Under a coordinate transformation $x'_i = \sum_j R_{ij}x_j$, the components of a second rank tensor change according to the rule $T'_{ij} = \sum_{k,l} R_{ik}R_{jl}T_{kl}$. Use this and the result of part (a) to verify the result of part (b).

Q 6) Find the leading term in the multipole exapansion of a disc carrying a surface charge density given by

$$\sigma(r) = \begin{cases} +\sigma_0 & \text{for } 0 \le r \le \frac{a}{\sqrt{2}} \\ -\sigma_0 & \text{for } \frac{a}{\sqrt{2}} < r \le a \end{cases}$$

Q 7) Consider three coplanar concentric rings of radii a, $2a$ and $3a$ carrying uniformly distributed charges $+5q$, $-8q$ and $+3q$ respectively.

a) Show that the quadrupole moment tensor vanishes for this system.

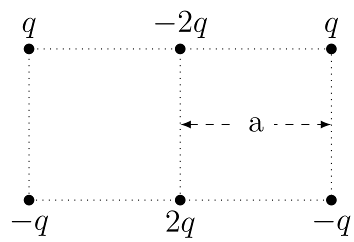
b) By simple high school physics, find the potential on the axis of the system.

c) Use this and the fact that the potential satisfies the Laplace equation everywhere for $r > 3a$ to determine the leading non-zero term of the potential.

d) Find the spherical multipole moments q_{lm} for $l = 3$ for this system and use this to calculate the l-th multipole contribution to the potential for $l = 3$. Show that this is the same term that you found in part (c) above.

Q 8) Find the leading term of the potential at $\vec{r}$ for $r \gg a$ for the two systems below

(i)



(ii)