PH3202 Problem Set 6

Q 1)a) In class we worked out the problem of a dielectric sphere placed in an otherwise uniform external electric field $\vec{E}_0$. You may start with the potential that we obtained as a result of the calculation and answer the following:

a) Determine the polarization vector field.

b) Hence determine the bound volume and surface charge densities.

c) Calculate the net charge and higher multipole moments of the bound charge distribution.

d) Directly calculate the potential due to the bound charge distribution, both inside and outside the sphere.

Q 2) Consider the problem of two semi-infinite dielectric materials of permittivity ϵ_1 and ϵ_2 respectively separated by a flat infinite plane with a free charge q embedded within the second medium at a distance d from the interface. We argued in class that this can be solved by setting up an image problem where//
(i) the potential in the second medium is given by the original free charge q and a charge q'at a distance d on the other side of the interface.
(ii) the potential ion the first medium is given by a single charge q'' at the same position as the original charge.

a) By imposing the continuity conditions, find out the values of q' and q''.

b) Find the bound surface charge density at the interface by (i) first calculating the polarization, and (ii) using the discontinuity in the normal component of the electric field.

c) What is the bound volume charge density?

d) Calculate the field due to the bound surface charge density and use it to explain the behavior of the electric field that you obtain from the image problem.

Q 3) Consider a dielectric sphere of radius a with a point charge $+q$ at a distance d from the center of gthe sphere with $d > a$.

a) Argue that the potential is given by

$$V(r,\theta) = \begin{cases} \sum_{l=0}^{\infty} A_l r^l P_l(cos)\,\theta & \text{for } r < a \\ \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(cos)\,\theta + \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{r^2+d^2-2rd\cos\theta}} & \text{for } r > a \end{cases}$$

b) Use the boundary conditions to determine the values of A_l and B_l and hence determine the potential everywhere.

c) Determine the bound charge density on the surface of the dielectric sphere.

d) Try to recast your result in the form of approprate image charges, if possible.