A Magnetofrictional Model for the Solar Corona

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Introduction

The magnetic field dominated low-β solar corona self-evolves to minimize magnetic stresses in response to flux emergence and driving by the high-β photospheric and sub-photospheric plasma. This dynamics lead to observed events like sigmoid, solar filament formation, solar flares, and coronal mass ejections [1] [2]. Shearing of magnetic field lines, magnetic reconnection, helicity constraints and magnetic relaxation dynamics all play important roles in these solar processes that create adverse space environmental conditions which define space weather. To understand the dynamics and evolution of magnetized plasma in the Sun's outer atmosphere we are developing a magnetofrictional model [3] [4] for the solar corona. The magnetofriction approach assumes the plasma velocity ($\mathbf{v}$) is governed by the Lorentz force, only, in the low-β regime of the corona. The effective frictional coefficient (ν) is a result of the assumption that a fictitious force acts in the background to neutralize the plasma motion. Here we present the model description and some initial results of a simulation in a small box which approximates a small portion of the solar atmosphere containing a solar active region magnetic structure.

Magnetofrictional Approach

We solve the magnetic induction equation on a spherical grid (r, θ, ϕ) such that r is the radial distance from the center of the sun, θ is the co-latitude and ϕ is the azimuthal angle. The basic assumptions in our model are :

1. The coronal field is in equilibrium in the time scale of large scale surface motions.
2. In the solar corona, magnetic forces dominate over the plasma forces ($\beta \ll 1$).
3. The magnetic field is considered as a mean field.

With these assumptions, the magnetic induction equation is solved for getting the evolution of the magnetic field in the global solar corona. The induction equation in terms of vector potential ($\mathbf{A}$) is given as,

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{v} \times \mathbf{B} - \eta_c \mathbf{j} \tag{1}$$

where the velocity and diffusivity are defined as,

$$\mathbf{v} = \frac{1}{\nu}\frac{\mathbf{j} \times \mathbf{B}}{B^2} + v_0 e^{-(2.5R_\odot - r)/r_w}\hat{\mathbf{r}}$$

$$\eta_c = \eta_0 \left(1 + 0.2\frac{|\mathbf{j}|}{|\mathbf{B}|}\right)$$

Here $\mathbf{A}$ is the magnetic vector potential, ν is the magnetofrictional coefficient. The second term in the velocity profile mimics the solar wind. The diffusivity profile is inspired from Yeates and Mackay [1].

Computational domain and Boundary conditions

The evolution equation (1) is solved on a three dimensional (3D) domain representing the solar corona in both hemispheres, where $R_\odot \leq r \leq 2.5R_\odot$, $60° \leq \theta \leq 100°$, and $150° \leq \phi \leq 210°$. We have used finite difference scheme. The time-stepping for our code is first-order euler method and slope calculations are second order accurate with central-difference. The grid spacing is uniform in this case with $dr = 0.015R_\odot$, $d\theta = 0.33°$ and $d\phi = 0.33°$. The number of grid points in each direction is $(101, 121, 181)$.

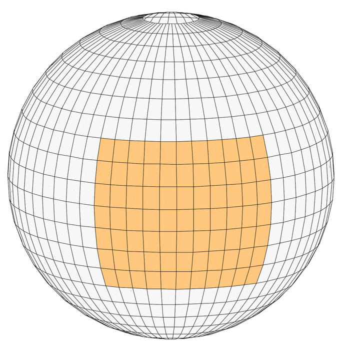
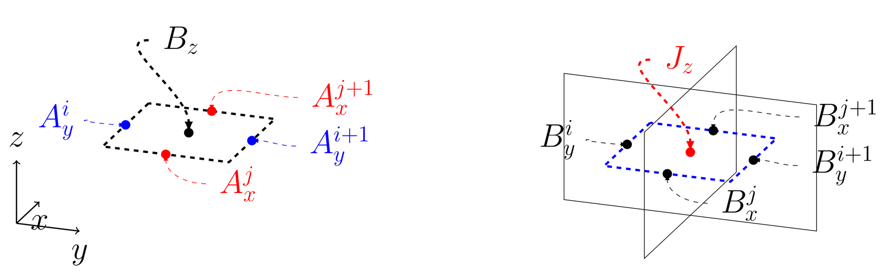


Figure 1: The shaded region shows the lower boundary of our simulation box on a scaled sphere.



The variables are defined on a staggered grid to avoid odd-even decoupling. Vector potential ($\mathbf{A}$) is defined on the cell ribs while the magnetic field ($\mathbf{B}$) is defined on the cell faces and the velocity is defined on the vertices. For numerical stability, CFL condition is satisfied everywhere. This model uses Van-Leer slope limiter [5] for constraining the areas of sharp gradients. We have used the following boundary conditions for the model

1. The Sun being spherical, the longitudinal boundary is periodic.
2. At the latitudinal boundaries, we have $\theta = 60°$ and $\theta = 100°$, $B_\theta = 0$.
3. At the top radial boundary ($r = 2.5R_\odot$), field lines are open because of the solar wind component.

The magnetic vector potential ($\mathbf{A}$) is initialized using algebraic functions in cartesian-like coordinate system as below [1]. In this case the cartesian-like coordinate system and spherical coordinate system are related as $x = \phi$, $y = -ln(tan\theta/2)$ and $z = ln(r/R_\odot)$. Here the parameters B_0 (peak magnetic field strength), ρ_0 (half separation distance between poles of opposite polarity) and β (twist in the bipolar region) are adjusted to setup the input profile.

$$A_x = \beta B_0 e^{0.5} z e^{-2\xi}$$
$$A_y = \beta B_0 e^{0.5} \rho_0 e^{-\xi}$$
$$A_z = -\beta B_0 e^{0.5} x e^{-2\xi}$$

The basic algorithm is to start off with a single bipole in the computational grid and relax it to a force-free configuration. Using the stress free configuration as input, evolve the active region further to study the dynamics in response to photospheric driving.

Force-Free Evolution

Here we run the simulation to study the evolution of a bipolar magnetic field configuration. The bipolar configuration is allowed to relax towards a force-free equilibrium in a low-β environment.

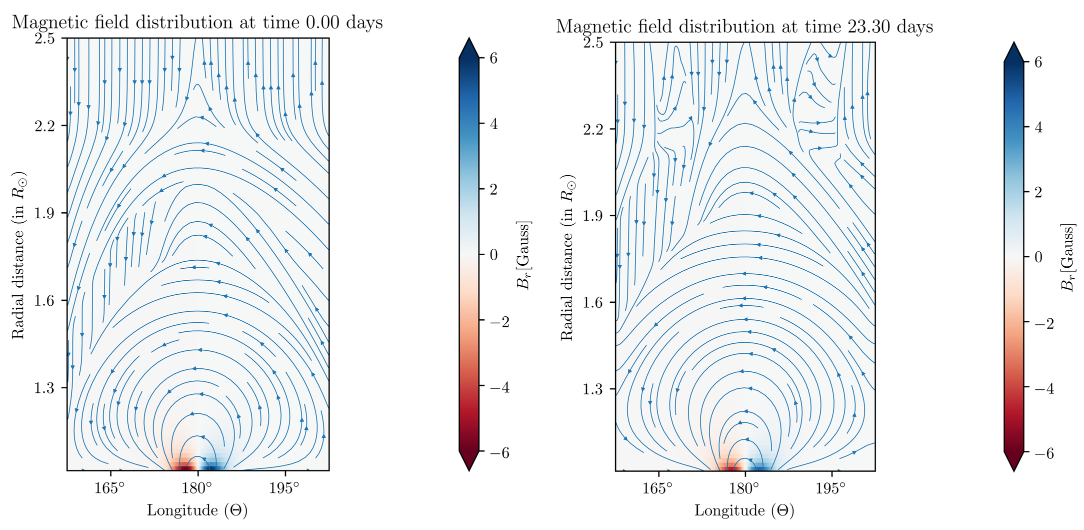


Figure 2: Relaxation of the bipole is shown along a $r - \theta$ plane at longitude $\phi = 80°$

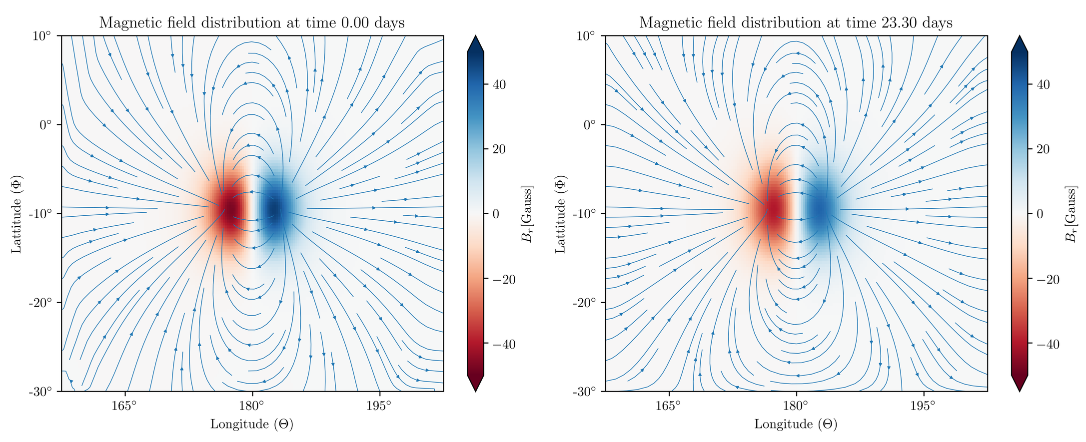


Figure 3: Relaxation of the bipole is shown along a $\phi - \theta$ plane at $1.015R_\odot$

Relaxation in response to Photospheric motion

Using the force-free state of the bipole as input for the simulation, we evolve the bipole in response to a longitudinal velocity profile. Due to the periodicity along longitudinal boundaries, the bipole comes back into the computational domain, which is clear from Figure- 4 & 5. Due to relaxation of field lines in a magnetic environment, the current and magnetic energy gets amplified in this process which is evident from Figure- 6.

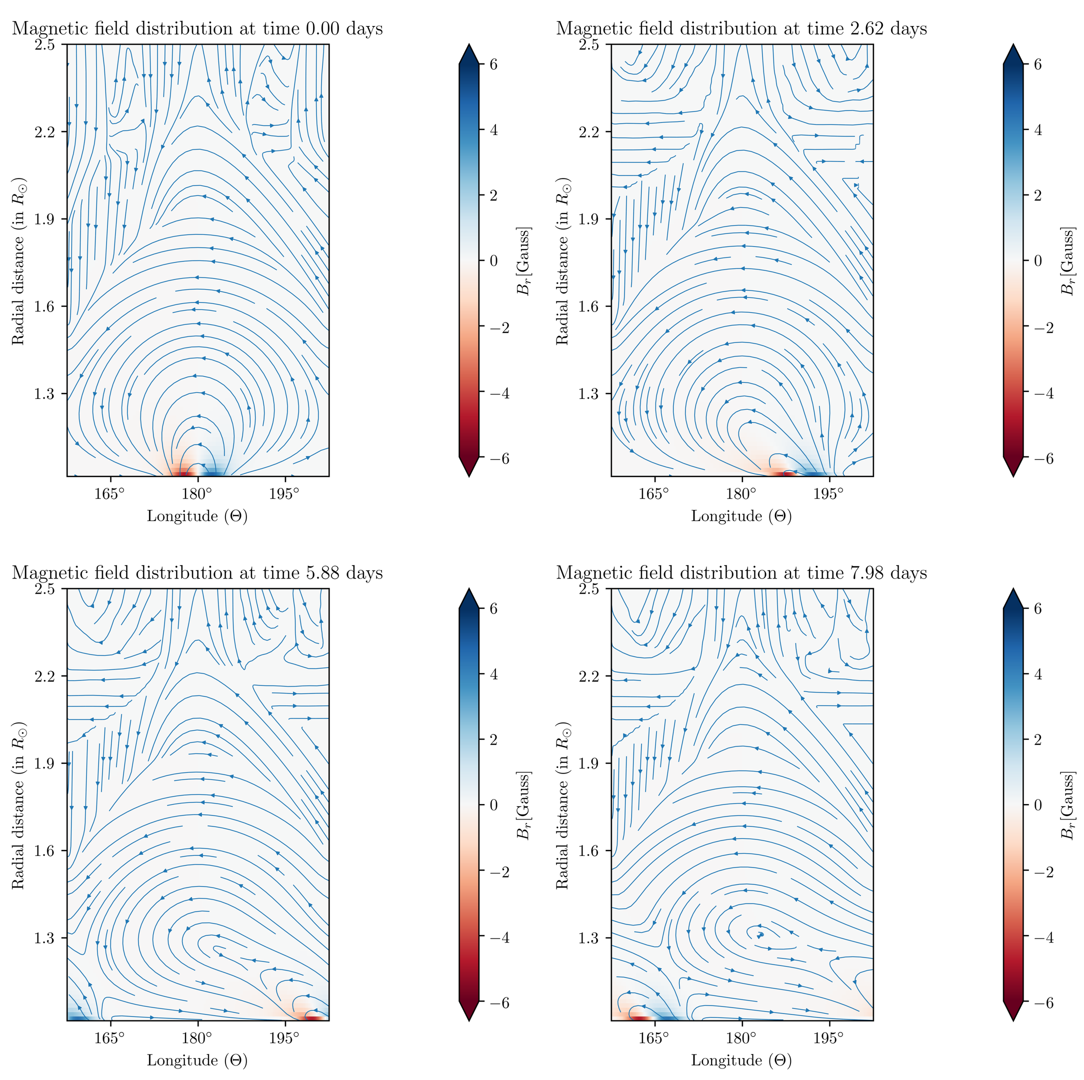


Figure 4: Evolution of the bipole is shown along a $r - \theta$ plane at longitude $\phi = 80°$

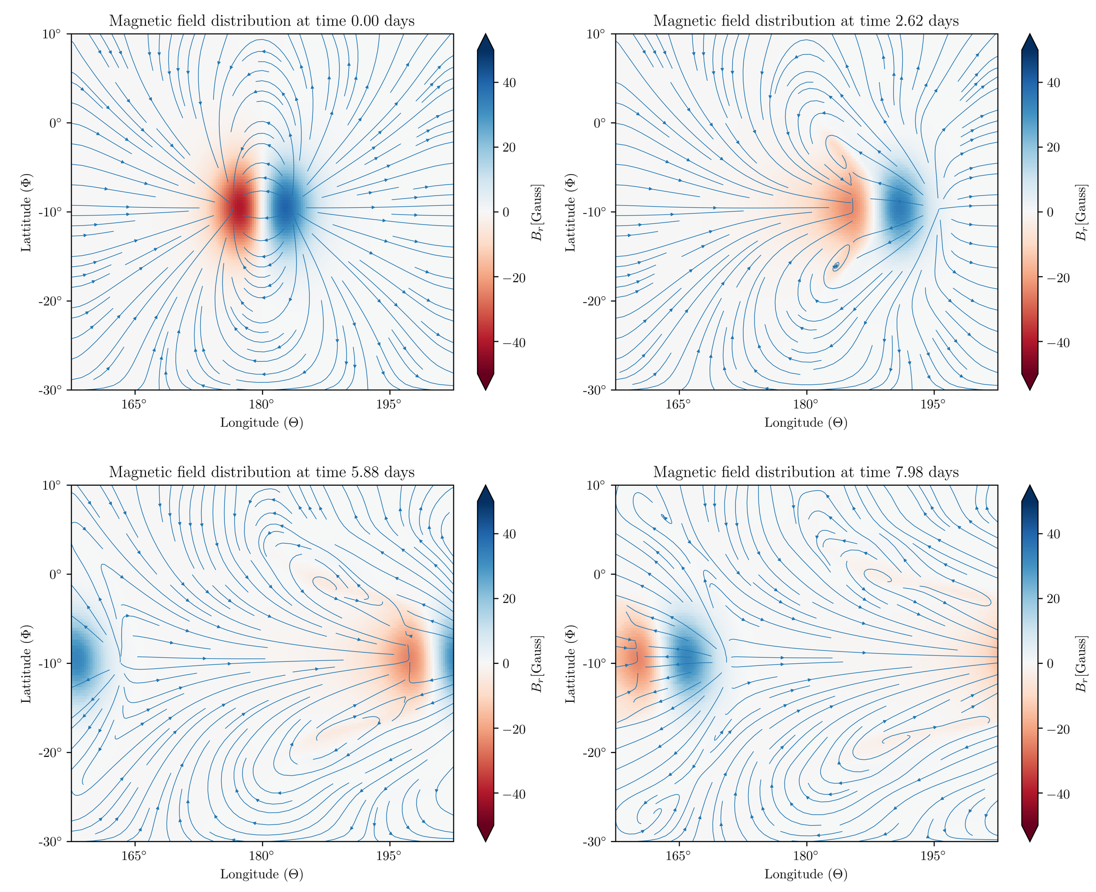


Figure 5: Evolution of the bipole is shown along a $\phi - \theta$ plane at $1.015R_\odot$

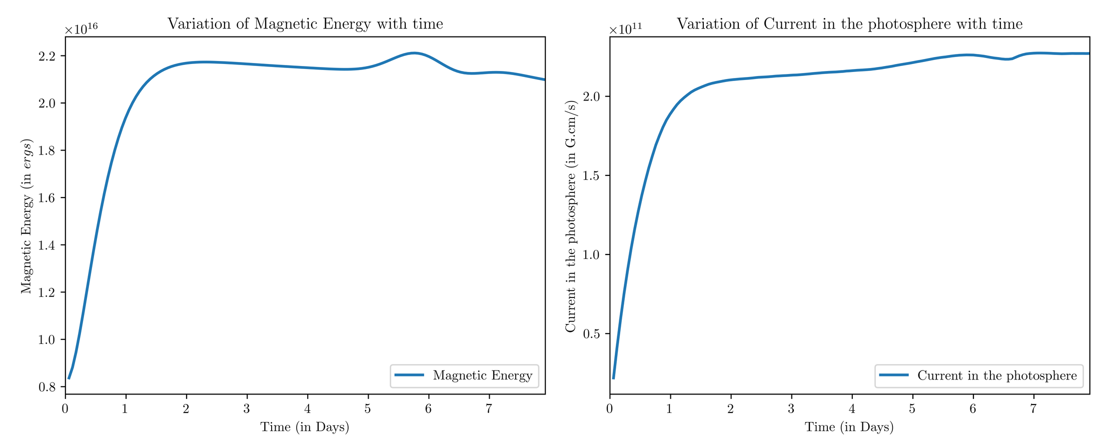


Figure 6: Variation of Current in the photosphere and Magnetic energy in the simulation domain during the simulation.

Future Work

Apart from simulating with synthetic data, we intend to make our model data-driven. Currently, the model uses multi-threading parallelization scheme. We plan to parallelize the model using OpenMPI to run more efficiently on a full scale cluster. We also plan to carry out global simulations of solar corona for significant fraction of a solar cycle.

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