Unification of forces

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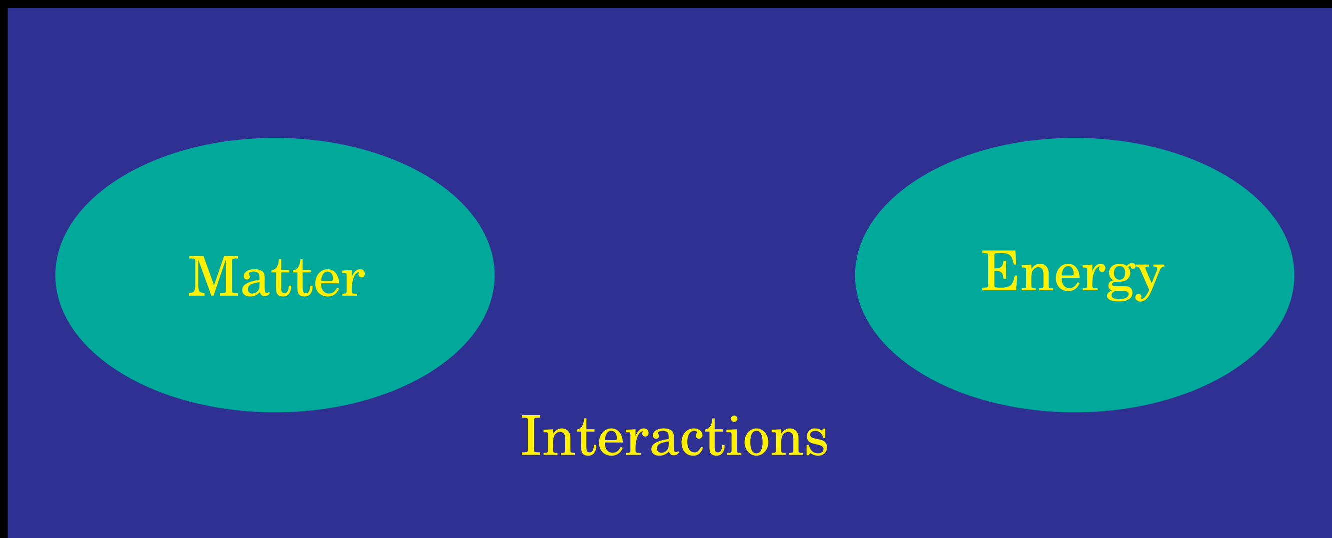
A simplified summary of the universe



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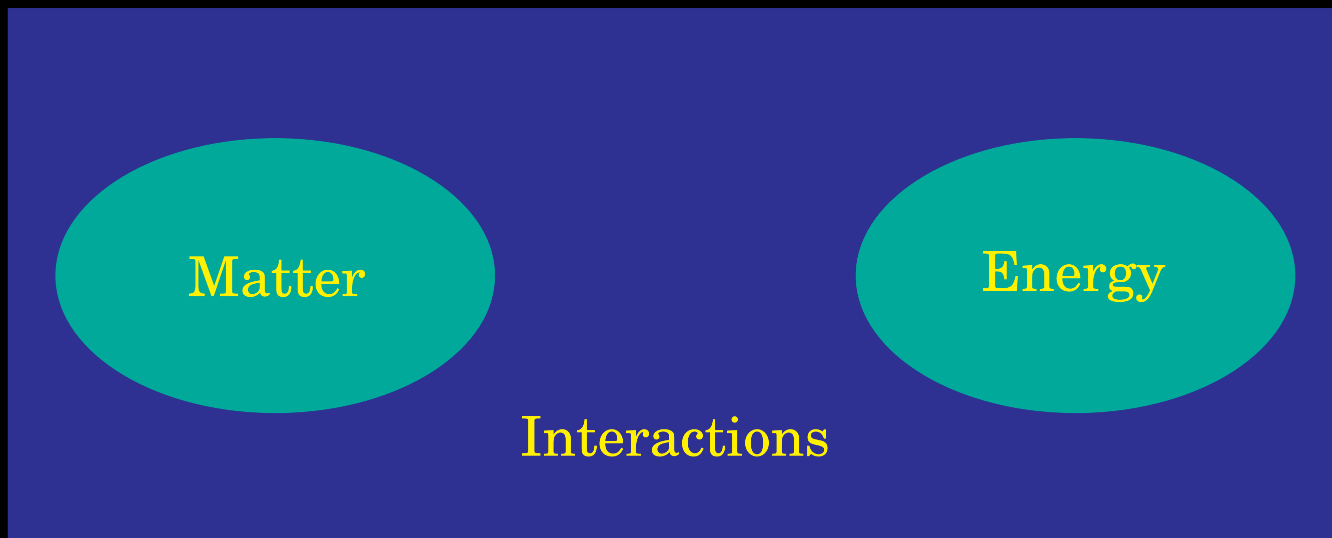

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Types of interaction:

Matter + Matter | Matter + Energy | Energy + Energy

Early theories: before unification

There are many kinds of matter and many kinds of energy.

Early theories tried to explain a single kind of phenomena involving a limited set of objects.

- Law of buoyancy
- Law of falling bodies
- Kepler's laws for planetary motion
- Law of reflection of light
- ...

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Newtonian revolution: two great aspects.

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The law of universal gravitation: As if the interactions have their own identity, e.g., through the gravitational constant which is not the property of any material object.

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Maxwell (1864): Unified laws of electricity and magnetism.

- ♣ The laws cannot be separated into an electric and a magnetic part.
- ♣ Since then, electromagnetism.
- ♣ Electromagnetic information travels at the speed of light.
- ♣ Light is a kind of electromagnetic wave.

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In 1915, Einstein provided a field theory of gravitation (General theory of relativity), at par with Maxwell's field theory of electromagnetism.

This theory treats gravitation as a consequence of the geometry of speacetime.

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Would it be possible to combine the two by using a general rank-2 tensor?

Einstein tried this in the final few decades of his life, without success.

Four kinds of interaction

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1. Gravitational : planetary system, stars and galaxies.
2. Electromagnetic : formation of atoms.
3. Strong : stability of atomic nuclei.
4. Weak : beta decay.

For the last two, one can have only quantum theory since they operate only at very short distances.

Quantum theory

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Energy exchanges in fields can be seen as exchange of particles which are quanta of fields. Any particle is the quantum of some field. Examples:

Field	particle	spin	type
Electron field	electron	$\frac{1}{2}$	fermion
EM field	photon	1	boson

Quantum electrodynamics: a U(1) theory

In classical electrodynamics, one can define vector and scalar potentials through the relations

$$\vec{B} = \vec{\nabla} \times \vec{A}\,, \qquad \vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\varphi\,.$$

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Since an expression of the form $e^{i\theta}$ can be thought of as a 1×1 dimensional unitary matrix, the symmetry is called a U(1) symmetry.

Kałuza-Klein approach to unification

Suppose the spacetime geometry is 5-dimensional.

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Question: Can the strong and weak forces be seen as geometrical effects?

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Examples of symmetry:

SU(2): Phases of two quantities, and rotation between the two.

SU(3): Phases of three quantities, and rotation between them.

The standard model

Strong, weak and electromagnetic — all three interactions are described through gauge theories.

Interaction	Gauge symmetry	Name	Gauge bosons
Strong	SU(3)	Quantum Chromodynamics	8 gluons
Weak and EM	SU(2)×U(1)	electroweak theory	W^+, W^-, Z, photon

This is also a unification of sorts, viz., all three interactions are described through the same kind of mathematical structure.

Grand unification

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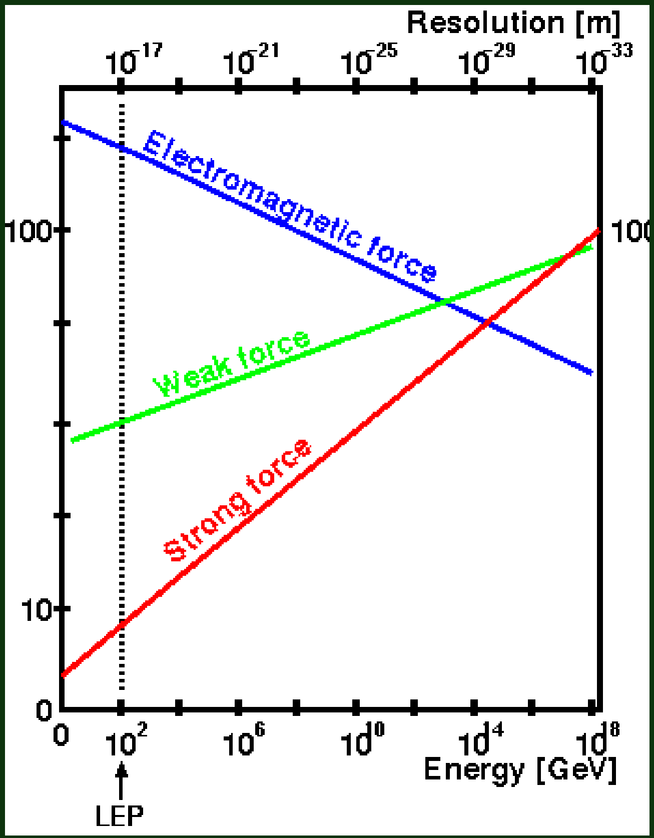
As energies get higher, strength of strong interaction decreases and strength of EM interaction increases.

They will meet at some point. If the weak interaction strength also meets there, then all three are the same, and SU(3), SU(2) and U(1) all can be parts of a bigger symmetry.

This idea is called grand unification. No experimental support for this idea yet, but it is a strong contender for unification.

Supersymmetry

The three lines do not in fact meet, unless there are other particles, unknown to present experiments.



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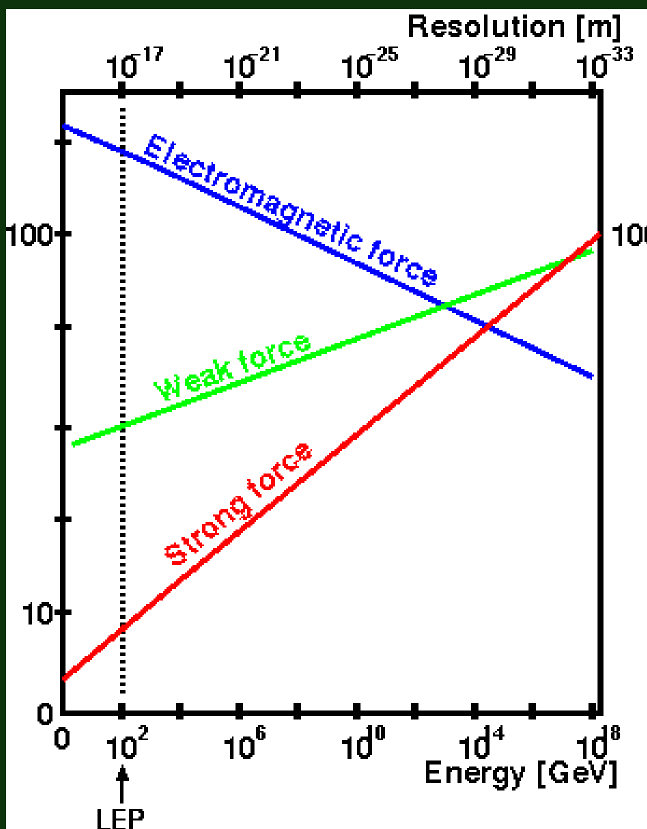
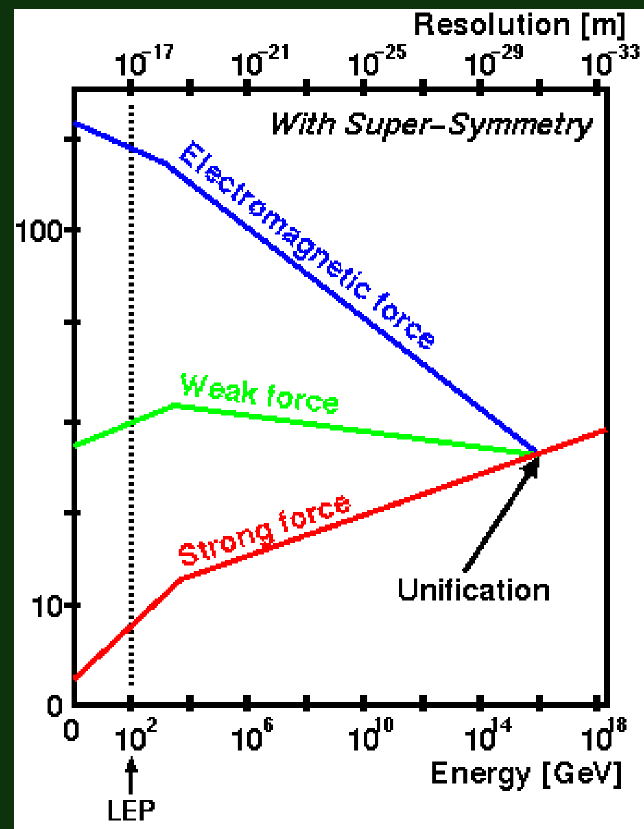
Supersymmetry: For every known fermion, there is a superpartner boson; for every known boson, there is a superpartner fermion.





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However, some features of the quantum theory of gravity are known.

- Particle mediating gravitational interaction (named graviton) has spin 2.

  Compare with: EM interactions are mediated by photons, whose spin is 1.
- Graviton is massless, just like the photon. That is why gravitational interaction has infinite range.

Superstrings

Fundamental objects are strings (1-dimensional objects), not particles.

Particles are characteristics of waves on these strings.

Waves on a normal string can only be bosonic. So add supersymmetry to include fermions. The result is called superstrings.

Success of the theory: the particle spectrum includes a massless spin-2 particle. This is the graviton. So gravitational interactions are automatically included in superstring dynamics.

Problems:

- The theory is consistent only in 10 (i.e., 9+1) dimensions.

  Solution: Use compactified dimensions (the old Kałuza-Klein idea).
- Where are the other interactions?