HOW WELL DO WE KNOW THE GRAVITATIONAL CONSTANT?

Sayan Kar

Dept. of Physics
&
CTS
IIT Kharagpur

KNOWING G WELL?

- **Measurements of G.**

- **Variation in time and space.**

- **G in higher dimensions.**

THE GRAVITATIONAL CONSTANT, G

Newton's 1687 law (in todays form)

$$\vec{F} = -\frac{GMm}{r^2}\,\hat{r}$$

Robert Hooke also knew!

$$[G] = M^{-1}L^3T^{-2}$$

- Newton never mentioned G in his Principia. He mainly worked with ratios.

- In 1798, Henry Cavendish measured the density of the Earth from which one can infer the value of G.

- G as a universal constant was first mentioned with the name f in 1873 by Alfred Cornu and Baptistin Baile.

- In 1894 Charles V. Boys first mentioned G as the universal Newtonian constant of gravitation in a paper in Nature.

THE VALUE OF G

(CODATA 2014).

$$G = 6.67408(31) \times 10^{-11}\ m^3/kg.s^2$$

Standard uncertainty in G:
$0.00031 \times 10^{-11}\ m^3 kg^{-1} s^{-2}$.

- **Standard uncertainty in Planck's constant** h **is** $0.000000081 \times 10^{-34}\ J.s$.

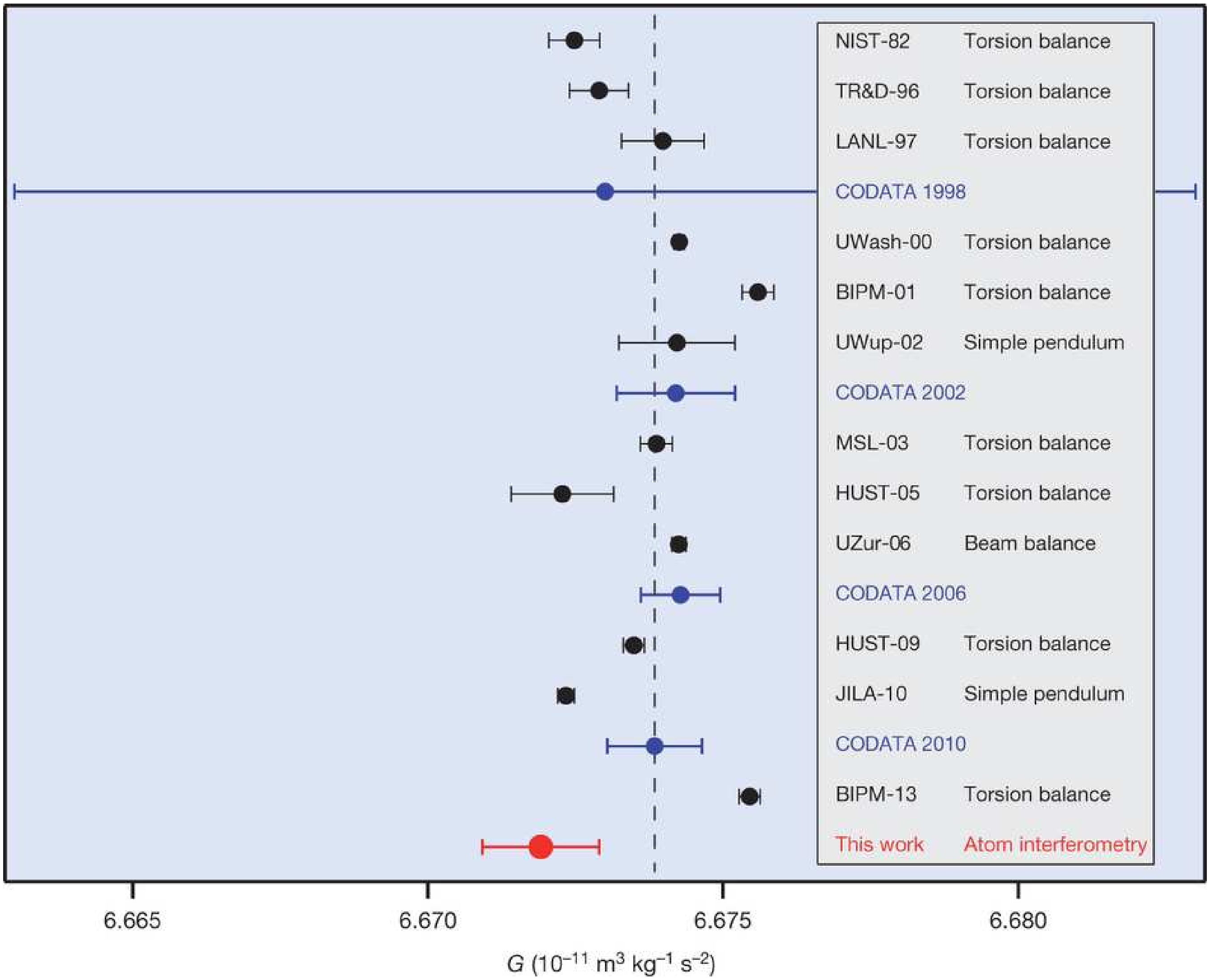
- **The speed of light in vacuum, is exact**

$$c = 299792458\ ms^{-1}$$

since length of a metre and the standard of time are fixed by using it.

- **G is known only upto three decimal places. Far worse than any other fundamental constant (say,** c **or** $\hbar$**).**

MEASURING G: SUMMARY



From 'Precision measurement of the Newtonian gravitational constant using cold atoms' by Rosi et. al., Nature 510, 518521 (26 June 2014)

CAVENDISH 1798

- The geologist John Michell (1724-1793) developed the torsion balance set-up.

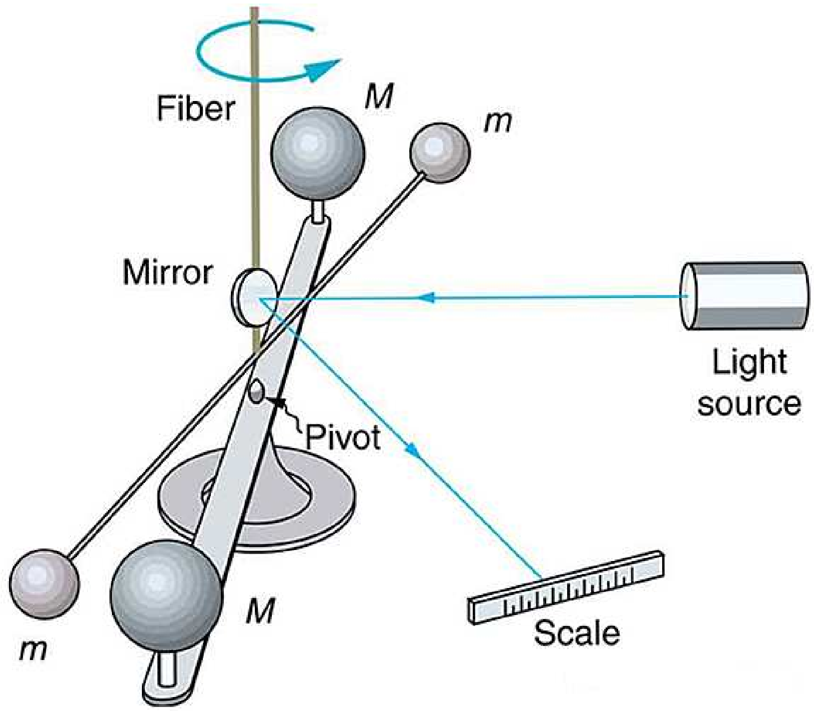
- Cavendish inherited it from Wollaston around 1797 and made minor modifications.

- Cavendish measured the density of the Earth with this apparatus.

- Cavendish's value of the density was 5.45 times that of water. The value we know of, today is 5.518. Surely, Cavendish was quite close.

- Many scientists measured the density of Earth after Cavendish. Reich, Baily, Cornu and Baille, Boys. Boys found a value of 5.53 in 1895.

- **The basic principle (in todays language).**



$$\kappa\theta = LF = L\frac{GMm}{r^2},\ T = 2\pi\sqrt{\frac{mL^2}{2\kappa}}$$

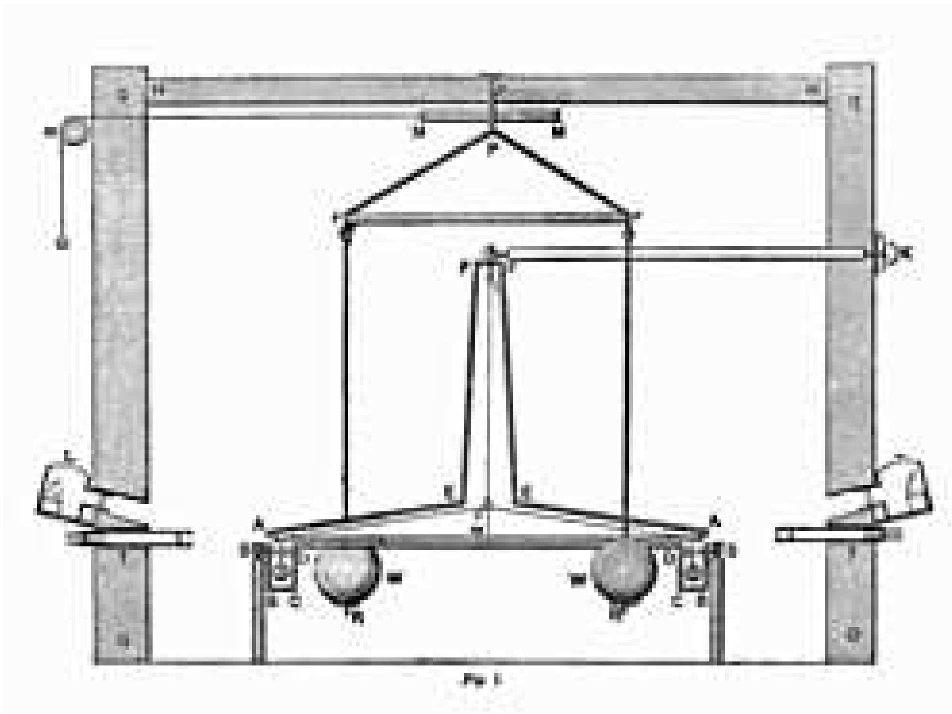
$$G = \frac{2\pi^2 L r^2}{MT^2}\theta,\ \rho_{earth} = \frac{3g}{4\pi R_E G}$$

$$\rho_{earth} = \frac{3MT^2 g}{8\pi^3 R_{earth} L r^2}\frac{1}{\theta}$$

Cavendish measured θ, T, r and found ρ_{earth}.

From his data $G = 6.74 \times 10^{-11}$ units.

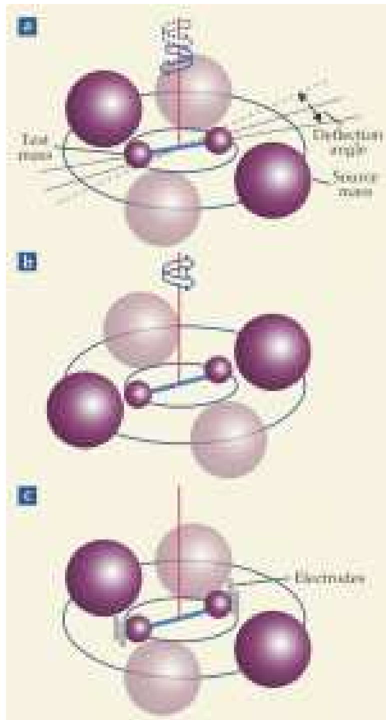
• **The Michell-Cavendish set-up**



A torsion balance made of a six-foot (1.8 m) wooden rod suspended from a wire, with a 2-inch (51 mm) diameter 1.61-pound (0.73 kg) lead sphere attached to each end. Two 12-inch (300 mm) 348-pound (158 kg) lead balls were located near the smaller balls, about 9 inches (230 mm) away, and held in place with a separate suspension system. The experiment measured the faint gravitational attraction (order of 10^{-7} Newton) between the small and large balls. Set up kept inside a box with a hole and Cavendish watched with telescope through the hole.

OTHER METHODS

(a) Usual Michell-Cavendish torsion balance



(b) Time-of-swing experiments, G calculated from change in oscillation period when source masses are repositioned between arrangements lying along (dark spheres) and orthogonal to (light spheres) the resting test-mass axis.

(c) The electrostatic servo-control technique. The gravitational force is calculated from the voltage that must be applied to nearby electrodes to hold the test assembly in place.

ATOM INTERFEROMETRY

• Atom interferometers use the wave nature of atoms. Measures phase difference between matter waves traversing different paths and meeting again.

• How do atom interferometers help in measuring G? Why not usual optical interferometers?

• Think of Young's double slit. When you calculate the phase difference between two paths you just do geometry, or, at best include the optical path.

• With matter waves one must calculate the phase difference using the classical action ($S = \int L(x, \dot{x}, t)dt = \int (\frac{1}{2}m\dot{x}^2 - V(x))dt$). The gravitational potential can appear in $V(x)$.

• Atoms, unlike light, are massive and bear gravitational signals in their interference patterns.

• One easily calculates $\Delta\Phi \sim gT^2$. Hence one can find g or G. This is the Kasevich-Chu interferometer!

• Wavelengths of matter waves typically 100 to 1000 times smaller than that of visible light. How to make beam splitters and mirrors for such matter waves?

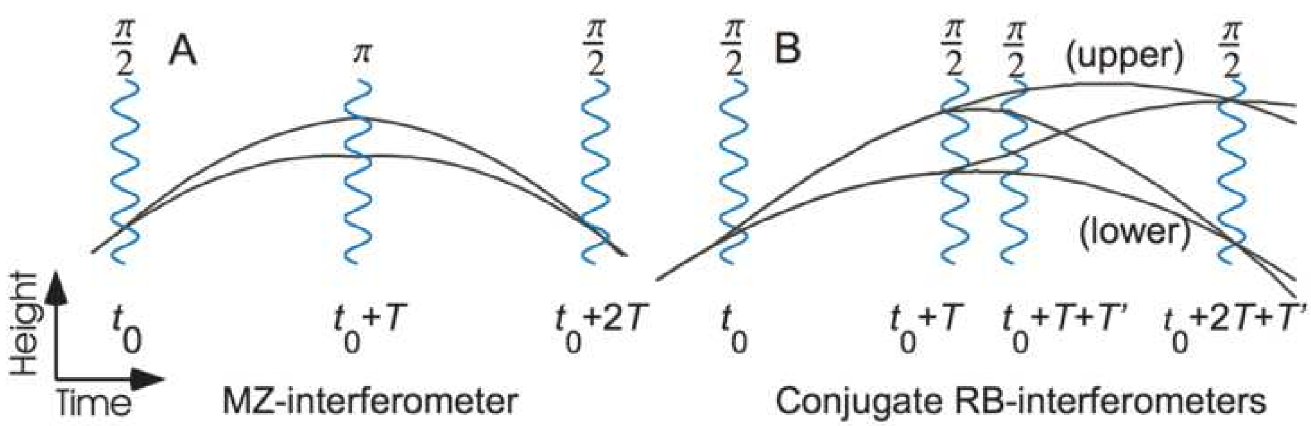
• Use light pulses which can work as beam splitters and mirrors for matter waves in specific quantum states.

• Light pulse atom interferometry.

BASIC SCHEME OF AI

• In atom interferometry, one starts with clouds of atoms laser-cooled to millionths of a degree above absolute zero.

• With pulses of light, one drives the atoms into quantum superpositions of having been kicked with the momentum of photons and not having been kicked.



• By manipulating the state of the atoms using light pulses, one steers the matter waves' paths and recombines the matter waves at the end of the experiment.

• The interference signal manifests as a population difference between final momentum states.

• Latest G value (atom interferometry):

G = 6.67191(99)$\times 10^{-11} m^3 kg^{-1} s^{-2}$ with a relative uncertainty of 150 parts per million.

Precision measurement of the Newtonian gravitational constant using cold atoms by G. Rosi, F. Sorrentino, L. Cacciapuoti, M. Prevedelli, G. M. Tino Nature, 510, 518521 (26 June 2014).

DEEP SPACE MEASUREMENT (2016)?

• A recent suggestion (Feldman et.al. CQG 2016) of a deep space measurement using the gravity train idea.

• Originally due to Robert Hooke, the gravity train is a hypothetical idea.

• Two antipodal points (say Hamilton(NZ) and Cordoba!) connected via a tunnel through the centre of the Earth.

• Motion in the tunnel that of a harmonic oscillator. $T = 2\pi\sqrt{\frac{R^3}{MG}}$.

• Drop a particle and it can reach the other end in 42 minutes (AJP, Cooper (1966$_)$!

• More recent calculation using Earth density variation gives 38 minutes (AJP Klotz (2015)).

Schematic:

• ‘Train’ is a small retroreflector which moves in a tunnel in the smaller sphere. It returns guided range pulses from a host spacecraft for measurements of the round trip travel time as noted in a stable clock on the host.

• Use the light-time measurements to build a profile of the position of the retroreflector and then find the time period. From this time period determine G.

• Multiple and more accurate (three orders better) determinations possible. Doable outside the Solar System, relative vacuum.

DOES G VARY?

• Weyl, Eddington had some ideas about large numbers.

• Dirac's observation in 1937:

The ratio of the electric and the gravitational force between two electrons is nearly the same as the age of the Universe in atomic units.

$$\frac{e^2}{4\pi\epsilon_0 G m_p m_e} \sim t\frac{4\pi\epsilon_0 m_e c^3}{e^2} \sim 10^{40} = N$$

• Dirac's Large Numbers Hypothesis states:

N is very large today because N has been increasing for a very long time.

How does G variation enter?

$$\frac{e^2}{4\pi\epsilon_0 G m_p m_e} \sim t\frac{4\pi\epsilon_0 m_e c^3}{e^2} \sim 10^{40} = N$$

- **Dirac suggests that the parameters in the equation have been changing in such a way as to preserve rough equality of the two ratios.**

- **Choosing units such that the atomic parameters are almost constant, preservation of the approx. numerical agreement of the ratios requires that as the universe expands, and t increases, the strength of the gravitational interaction decreases, $G \sim t^{-1}$.**

- **First mention of a varying G.**

VARYING G THEORIES

• In 1961 the Jordan-Brans-Dicke theory of gravity was proposed with a spacetime varying G. A scalar field $\Phi(t, \vec{x})$ did the job. Brans-Dicke theory had problems in observational verification and was discarded.

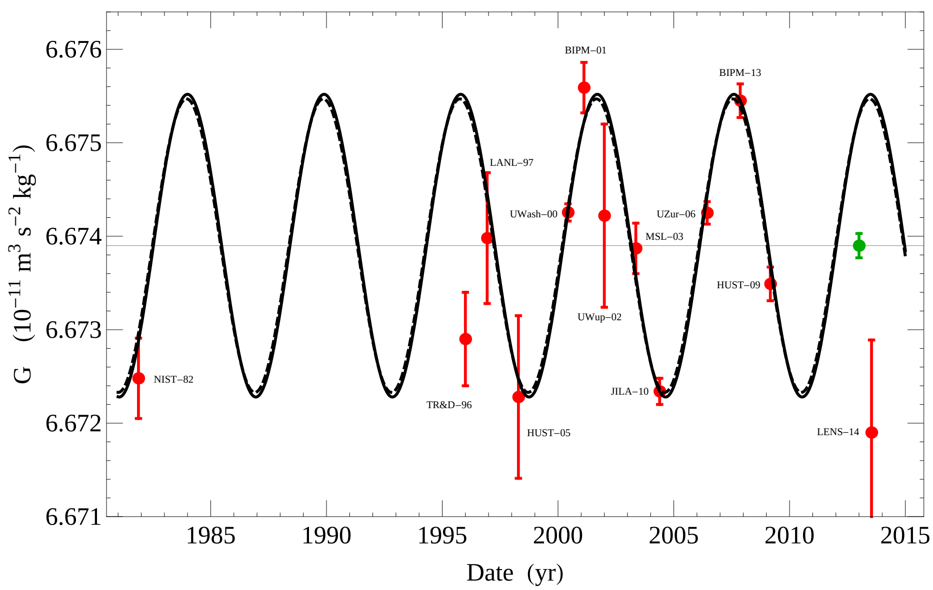
• In the 1980s versions of low energy, effective superstring theory arrived and were found resemble Brans Dicke theory. They predict a varying G, through a spacetime dependent scalar field $\Phi(t, \vec{x})$, the dilaton.

• More recently, effective 4D theories arising from higher dimensions also end up being like a Brans-Dicke theory.

• In full string theory the gravitational constant (in higher dimensions) is related to the string length ($\sqrt{\alpha'}$) and the coupling constant g for closed string interactions.

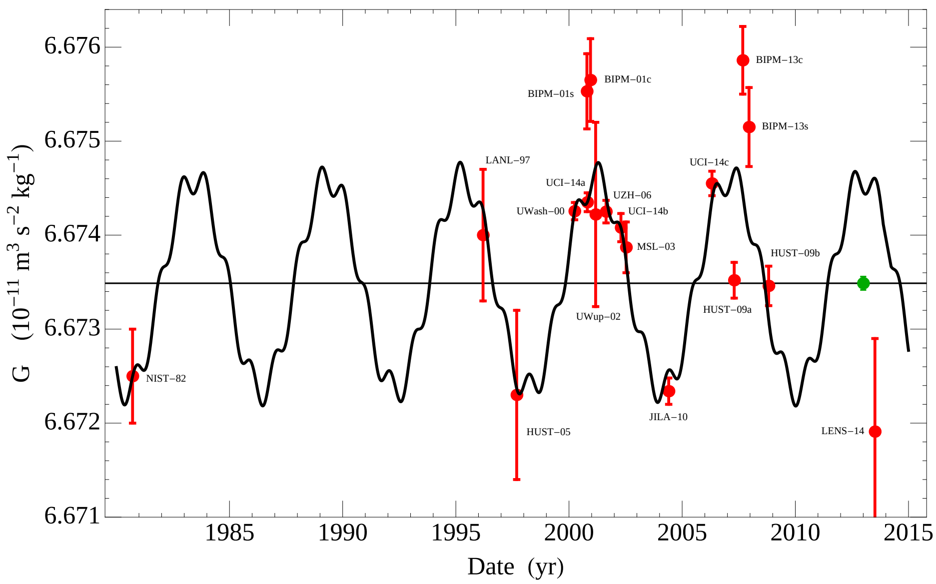
A RECENT CURIOSITY

- Use the set of about **12 G measurements since 1962**. Plot as a function of time. Anderson, Schubert, Trimble, Feldman (EPL(2015))



- The values for G are **oscillatory** in nature, with a period of $P = 5.8990.062yr$, **amplitude** $(1.6190.103)10^{-14}$ m^3 kg^1 s^2, and mean-value crossings in 1994 and 1997.

• The only measurement with the same period and phase is the Length of Day (LOD - defined as a frequency measurement such that a positive increase in LOD values means slower Earth rotation rates and therefore longer days.



• Two-period ($P_1 = 1.02$ yr, $P_2 = 5.9$ yr) fit to 18 revised G measurements recommended by Schlamminger, Gundlach and Newman (PRD(2015)). The revised weighted mean and its uncertainty is indicated by the green dot.The two-period model is no longer in phase with the LOD sine wave.

• **Possibly no fundamental physics?? Errors?? What errors and why??**

NEW ANALYSIS (2016)

• Pitkin (2015) using Bayesian model comparison, ar- gued that a model with an unknown Gaussian noise component is favored over any periodic variations.

• Desai (2016) using frequentist model comparison tests shows that a constant term along with an unknown systematic offset provides a better fit to the measurements of G.

RELATED COMMENT (2016)

• Unnikrishnan (2016) observed that the 5.9 year period in the length of the day of Earth, with amplitude 0.13 ms, matches in period and phase with Earth-Jupiter distance attaining an extremum, at those times when Jupiter is at its perihelion or aphelion and the Sun and Earth align along its orbital major axis.

21 YR PULSAR DATA

- **Neutron stars:** Compact stars resulting from gravitational collapse after a supernova explosion. Made of neutrons– degeneracy pressure. Radius 12-13 km, mass about twice the solar mass, density very high ($6-8\times 10^{17}\ kg/m^3$). Equation of state of matter not yet fully known.

- **Pulsar:** Highly magnetized, rotating neutron star which emits a beam of electromagnetic radiation. A binary pulsar is a pulsar with a binary companion, often a white dwarf or neutron star.

- **Pulsed emission:** The beam can be observed when it is pointing towards Earth, much the way a lighthouse can only be seen when the light is pointed in the direction of an observer. Thus, the pulsed appearance of emission.

- **Clocks:** Such neutron stars have short, regular rotational periods which produces a very precise interval between pulses that range roughly from milliseconds to seconds for an individual pulsar. Remarkable clocks, can compete with atomic clocks!

Zhu et. al, ApJ (2015)

- 21-yr timing of one of the most precise pulsars: PSR J1713+0747.

- Measured a change in the observed orbital period of PSR J1713+0747.

- Subtracting out changes due to other reasons, the intrinsic change in the orbital period, $\dot{P}^{Int} = -0.20 \pm 0.17\ ps\ s^{-1}$, is almost zero.

- This result, combined with measured $\dot{P}^{Int}$ of other pulsars, places limits on potential changes in G as predicted in alternative theories of gravitation.

- $\frac{\dot{G}}{G}$ is consistent with zero: $(-0.6 \pm 1.1) \times 10^{-12}\ yr^{-1}$, [95% CL]. This is the best $\frac{\dot{G}}{G}$ limit from pulsar binary systems.

THE THEORISTS' GAME

If there are more than three spatial dimensions, say n dimensions in all, then:

• Question 1: What happens to the gravitational constant?

• Question 2: What happens to the speed of light in vacuum?

• Question 3: What happens to Planck's constant?

We will analyse the first two questions on the basis of Newtonian, Maxwellian physics.

The third question has no answer. We assume nothing new happens.

However, see Agnese, La Camera and Recami, Black body radiation derived from a minimum knowledge of physics, Nuovo Cimento (1999)

THE 4D PLANCK SCALE

- **Using G, c and $\hbar$ one defines natural units and the Planck scale.**

- **Combinations with length (L) mass (M), time (T) dimensions exist.**

- **We have:**

$$l_p = \sqrt{\frac{G\hbar}{c^3}} = 1.616 \times 10^{-35} m$$

$$m_p = \sqrt{\frac{\hbar c}{G}} = 2.176 \times 10^{-8} kg$$

$$t_p = \sqrt{\frac{G\hbar}{c^5}} = 5.391 \times 10^{-44} s$$

Planck energy:

$$E_p = 1.2 \times 10^{19} GeV.$$

- **One also has Planck charge and Planck temperature.**

G and EXTRA DIMENSIONS

Poisson eqn. for gravity in arbitrary dimensions.

$$\nabla^2 \phi^{(n)} = 4\pi G_{(n)} \rho^{(n)}$$

$\phi^{(n)}$ is the gravitational potential, $G_{(n)}$ the gravitational constant and $\rho^{(n)}$ is the mass density in $3+n$ dimensions.

Note $[G_{(n)}]$ different for different n.

$$\left[G_{(n)}\right] = M^{-1} L^{n+3} T^{-2}$$

$\rho^{(n)} G_{(n)} \rightarrow$ dimension independent.

$$[\rho^{(n)} G_{(n)}] = T^{-2}$$

in all dimensions.

Law of gravity with n extra dimensions?

$$F_{(n)} = \frac{G_{(n)} m M}{r^{n+2}}$$

$$n = 0 : F_{(0)} = \frac{G_{(0)} m M}{r^{2}}$$

$$n = 1 : F_{(1)} = \frac{G_{(1)} m M}{r^{3}}$$

$$n = 2 : F_{(2)} = \frac{G_{(2)} m M}{r^{4}}$$

- **No dimensionless combination of $G_{(n)}$, $\hbar$, c exists. We have a Planck scale!**

$$l_p^{(n)} = \left(\frac{G_{(n)} \hbar}{c^3}\right)^{\frac{1}{n+2}}$$

$$m_p^{(n)} = \left(\frac{\hbar^{1+n} c^{1-n}}{G_{(n)}}\right)^{\frac{1}{n+2}}$$

$$t_p^{(n)} = \left(\frac{G_{(n)} \hbar}{c^{n+5}}\right)^{\frac{1}{n+2}}$$

We do not know $G_{(n)}$ so no values like in three spatial dimensions. Exploit this!

c and EXTRA DIMENSIONS

Poisson eqn. for electrostatics in **arbitrary** dimensions.

$$\nabla^2 \phi_E^{(n)} = -\frac{\rho^{(n)}}{\epsilon_0^{(n)}}$$

Poisson eqn. for magnetostatics in **arbitrary** dimensions.

$$\nabla^2 \vec{\mathbf{A}}^{(n)} = -\mu_0^{(n)} \vec{\mathbf{J}}$$

Like G, ϵ_0 and μ_0 are dimension dependent but their appearance in the numerator and denominator in the Poisson eqn, keeps c dimension independent. This happens because $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$.

- Thus, in a higher dimensional theory, one can still have the same $c = 3 \times 10^8 ms^{-1}$.

TRADE-OFFS!

Recall $\left[G_{(n)}\right] = M^{-1}L^{n+3}T^{-2}$

Assume:

$$G_{(n)} = R^n G_{(0)}$$

R **is the radius of the extra dimension.**

Then, we have,

$$F_{(n)} = \frac{R^n G_{(0)} mM}{r^{n+2}}$$

For example, for one extra dimension, we can have R **as the radius of a circle.**

With two extra dimensions, we can have R **as the radius of a sphere.**

and so on ....

Then, one can show, using $E_P^{(n)} = m_P^{(n)} c^2$,

$$\left(E_P^{(0)}\right)^2 = \left(\frac{R}{\hbar c}\right)^n \left(E_P^{(n)}\right)^{n+2}$$

If $E_P^{(0)} = 10^{19}$ **GeV**, and $E_P^{(2)} = 1TeV$, then $R \sim 2mm$ (size of extra dimensions).

Thus, the $n+4$ **dimensional Planck energy** can be chosen as **lower** in value (1 TeV) in the **presence of extra dimensions**. The **four dimensional Planck energy of 10^{19} GeV** then follows as a consequence.

We can have various other types of trade-offs (say, $G_{(n)} = f(k) G_{(0)}$). That follows from newer models some of which have **background curvature** (like the Randall–Sundrum set-up).

ARE THERE EXTRA DIMENSIONS?

• If we propose the existence extra dimensions, then we must find out where they are today.

• Not directly observable but signatures there.

• One possible route is

Compactification

But there are too many options and theories.

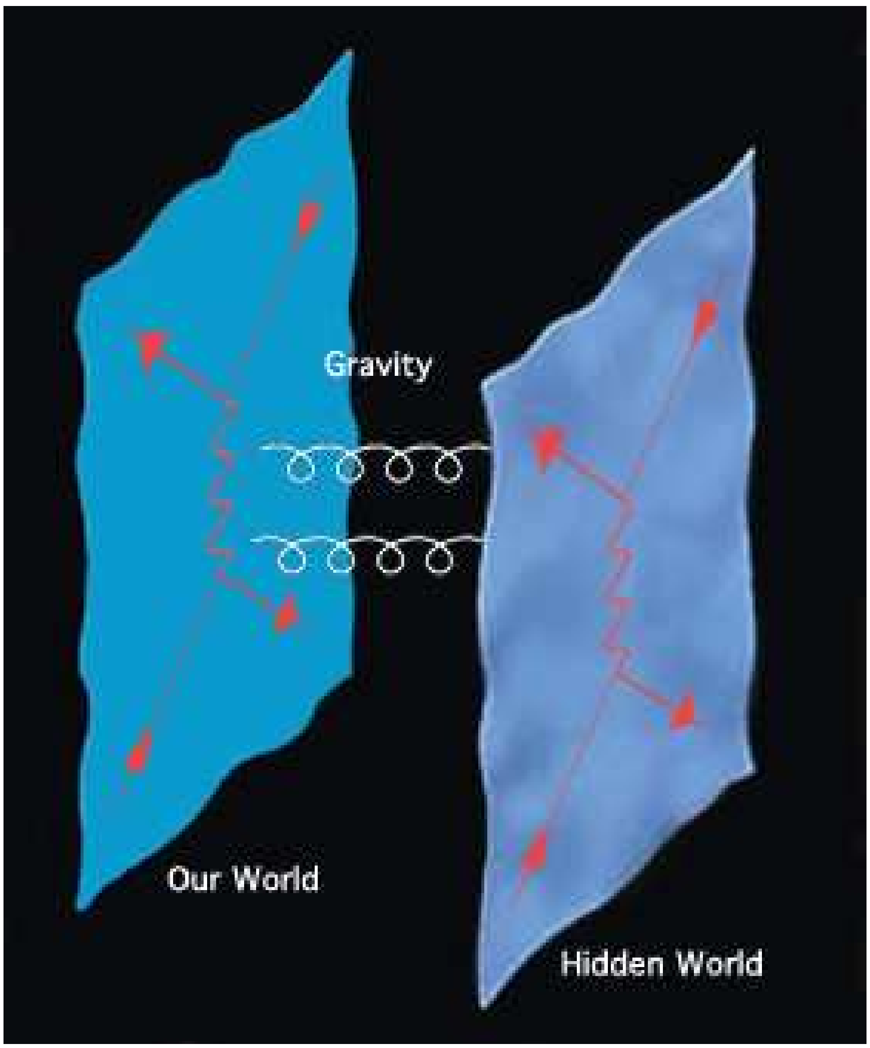
• An alternative to compactification is the

Braneworld

We need localisation of fields on the brane to happen.

BRANEWORLDS

• We live in a four dimensional world which is embedded in a higher dimensional space-time. We are oblivious of the existence of extra dimensions.



• If experiments show the presence of extra dimensions then we say that the 10^{19} GeV Planck energy arises only in the 4D world.

• Other mechanisms for alternative trade-offs exist.

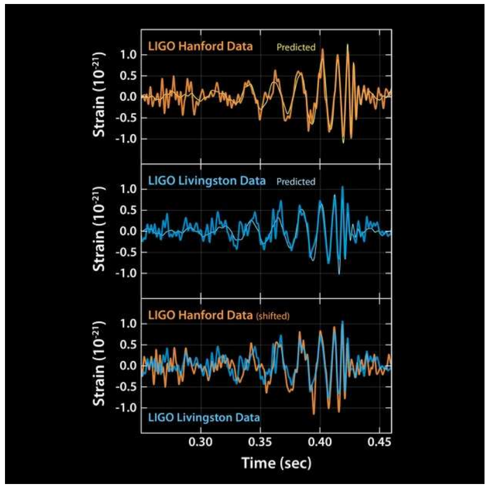
• Future accelerator experiments, are supposed to check for the presence of extra dimensions in proposed models.

• Such models are known as the braneworld models.

One can possibly 'see' Planck scale effects at TeV energies (maybe at LHC)!.

The experimental verification of extra dimensions would therefore tell us about the value of the gravitational constant in a higher dimensional spacetime.

VARYING G AND GRAVITATIONAL WAVES?



- GW150914, detected at Livingston and Hanford Michelson-Fabry-Perot detectors on September 14, 09:50:45 UTC.

- Change in arm length due to the signal, $\Delta L(t) = \delta L_x - \delta L_y = h(t)L$. $h(t)$ is plotted as a function of time. Very very small amplitude 10^{-21}.

- Signal is like a damped sinusoid.

BINARY BLACK HOLE MERGERS

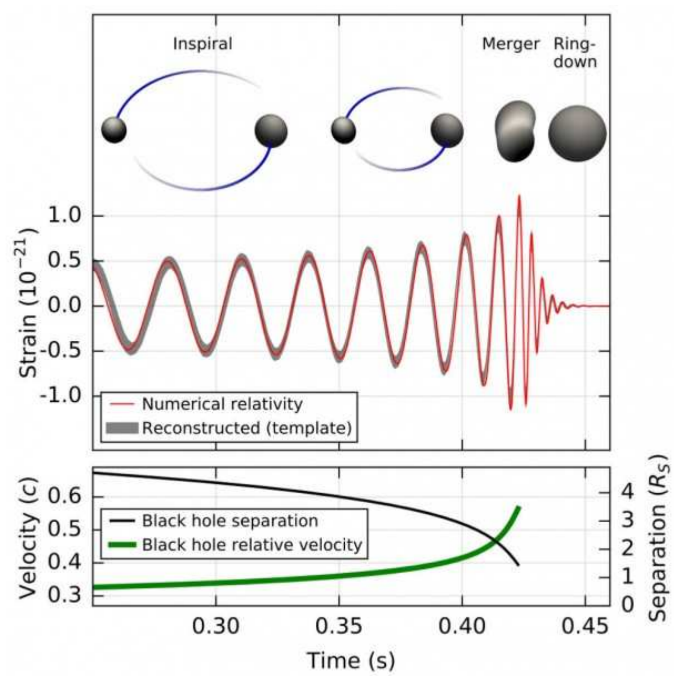
• An event occured 1.3 billion light years away. Two black holes collided and merged to form one black hole. The masses of the initial black holes together is greater than that of the final one. So there is a loss in terms of energy. This is the energy carried away in gravitational waves.

• Numerical Relativity can model the collision and its consequences give rise to the waveform at fixed location, i.e. $h(t)$. It is called a quasinormal mode (Vishveshwara, Chandrasekhar, Detweiler).

• This wave has traveled all the way and has progressively weakened.

• It hit the LIGO detectors at Hanford and Livingston on Sept.14, 2015.

BINARY BLACK HOLE MERGERS



• GW150914 is from a **binary black hole merger** with source frame masses 36 and 29 solar masses. Very good statistics.

• A **second signal GW151226** also detected on December 26, 2015. It is also from black holes 1.4 billion light years away.

VARYING G?

• **Theoretical physics implications of the binary black hole merger GW150914 by Yunes, Yagi and Pretorius, arxiv 1603.08955.**

GW150914 can place constraints on the size of the extra dimension and any time-variation in G, but these are worse than other current constraints, such as those imposed with binary pulsars. This is because these effects enter at 4PN order, which implies that binary pulsar observations and BH low-mass X-ray binaries lead to much stronger limits. Constraints that could be placed by space-borne detectors, such as eLISA and DECIGO, could be many orders of magnitude stronger than aLIGO (and competitive with current bounds). Once more, nonetheless, notice that the GW150914 constraints are unique in that they use data from the extreme gravity regime.

• **Wait for more and better GW observations.**

ANOTHER CONSTANT?

- In classical gravity, there are two fundamental constants, G and c. Is there room for a third, which can be used to define a length, time and mass?

- Motivation comes from Born-Infeld electrodynamics which removes the infinite energy at the electric charge location by introducing a new action and a new constant.

- In Born-Infeld gravity one introduces a third constant κ (dimension L^2). With G, c, κ one may define length, time and mass scales.

- One can also avoid the big-bang in such modified gravity.

• Born Infeld gravity inside matter (with spatial variation) has a different Newtonian limit which depends on κ. Therefore, G variation may be analysed from this perspective.

• Modified Newtonian Dynamics (MOND) changes the second law of Newton and brings in a new constant and a new function. Variation of G may be addressed in MOND (see Klein (2016)).

• One can also link up modified gravity, G variation with Galaxy Rotation Curves and the problem of dark matter.

A CURIOSITY

- Look at the dimensions of the constants.

$$[G] = L^3 M^{-1} T^{-2}$$

$$[c] = L T^{-1}$$

$$[\hbar] = L^2 M T^{-1}$$

$$[k_B] = L^2 M T^{-2} \Theta^{-1}$$

$$[\frac{1}{\epsilon_0}] = L^3 M T^{-2} Q^{-2}$$

Let $\Delta = a + b + c + d + e$ where a given constants' dimension is $M^a L^b T^c Q^d \Theta^e$.

For G, c, $\frac{1}{4\pi\epsilon_0}$ and k_B, $\Delta = 0$. For $\hbar$, we have $\Delta = 2$. To have a scale with formulae for length, mass etc. we must have $\Delta \neq 0$ for one constant. If not, a dimensionless combination will exist. $\Delta = 2$ and $\hbar$ is one choice.

- Is this an accident?

HOW WELL DO WE KNOW G?

• Measurements of G are still being done. Main problem is to make an isolated system. Measurements, as of now, are not very good.

• Confusions exist about G variations. No answer known.

• If extra dimensions exist, there is an extra-dimensional G. We need to worry about its value and its relation with the four dimensional G.

• Final word can only come from better experiments/observations on G measurements. Both LIGO and LHC can help.

AFTERWORD

- A nice and brief summary of G measurements and the future by Terry Quinn, Nature Physics 12, 196 (2016) has appeared.

- See also C. Speake and T. Quinn, The search for Newton's constant, Phys. Today 67, 27 (2014).

- NSF, USA has announced through its Ideas Lab the Measuring Big G Challenge.

- This is what they write inviting proposals:

We are looking for bold researchers willing to design and undertake new measurements of Newtons gravitational constant G (Big G). No background in the field? No problem!