ChernSimons-A New Self Dual GaugeTheory in 2+1

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What is it and why is it important

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- **Boundary Phenomenon in 3+1 Dimensions.**

Boundary Phenomenon

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For a finite sample, we write

$$S = \int d^4x \chi_{\boldsymbol{b}} \epsilon^{\alpha\beta\gamma\delta} \partial_\alpha A_\beta \partial_\gamma A_\delta \tag{3}$$

$$S = \int d^4x \chi_{\boldsymbol{b}} \partial_\alpha \left(\epsilon^{\alpha\beta\gamma\delta} A_\beta \partial_\gamma A_\delta \right) \tag{4}$$

Boundary Phenomenon Contd...

Now

$$\chi_{\boldsymbol{b}}\partial_\alpha\left(\epsilon^{\alpha\beta\gamma\delta}A_\beta\partial_\gamma A_\delta\right)-\partial_\alpha\left(\chi_{\boldsymbol{b}}\epsilon^{\alpha\beta\gamma\delta}A_\beta\partial_\gamma A_\delta\right)=-\left(\partial_\alpha\chi_{\boldsymbol{b}}\right)\left(\epsilon^{\alpha\beta\gamma\delta}A_\beta\partial_\gamma A_\delta\right) \tag{5}$$

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Lets assume the boundary is X-Y plane

$$\partial_\alpha \chi_b = \delta(z) \tag{6}$$

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So we have

$$-\left(\partial_\alpha\chi_{\boldsymbol{b}}\right)\left(\epsilon^{\alpha\beta\gamma\delta}A_\beta\partial_\gamma A_\delta\right)=\epsilon^{\beta\gamma\delta}A_\beta\partial_\gamma A_\delta \tag{7}$$

SSB generating CS term

In 2+1 D we have genralised definition of covariant derivative

$$D_\mu = (\partial_\mu - \imath e A_\mu - \imath g \epsilon_{\mu\nu\rho} A^\rho) \tag{8}$$

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Consider the Abelian Higgs Model

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (D_\mu \phi)^* (D^\mu \phi) - \alpha \left(|\phi|^2 - a^2 \right)^2 \tag{9}$$

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- Non zero VEV of scalar field can generate CS term $2egv^2 \epsilon_{\mu\nu\rho} A^\mu \partial^\nu A^\rho$.

Chern-Simons Lagrangian

$$\mathcal{L}_{CS} = \frac{\kappa}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho \tag{10}$$

- It is possible to write down such lagrangian only in odd dimensions!

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Deceptively Trivial Equation Of Motion:

$$F^{\mu\nu} = 0 \tag{12}$$

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$$\frac{\kappa}{2}\epsilon^{\mu\nu\rho}F^{\nu\rho} = J^{\mu} \tag{13}$$

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$$\Rightarrow \partial_{\mu}J^{\mu} = 0 \tag{15}$$

- Bianchi Identity is compatible with current conservation.

Coupling to Matter Field

Euler Lagrange equation can be recast in the following form

$$\rho = \kappa B \quad (16)$$

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- Anyonic exchange phase $\frac{e^2}{2\kappa}$

Solving the Constraint Equation

From $\kappa B = \kappa \epsilon^{ij} \partial_i A_j = \rho$ we get

$$\vec{A} = -\frac{1}{\kappa} \int d^2x' \vec{G}(\vec{x} - \vec{x'}) \rho(\vec{x'}) \tag{19}$$

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Hence

$$G_j = -\frac{1}{2\pi}\epsilon_{\alpha j}\partial^{\alpha} ln(r) \tag{21}$$

Topological Mass

$$\mathcal{L}_{MCS} = -\frac{1}{4e^2} F^{\mu\nu} F_{\mu\nu} + \frac{\kappa}{2} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho \tag{22}$$

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Corresponding EOM is

$$\partial_\alpha F^{\alpha\beta} + \frac{\kappa e^2}{2} \epsilon^{\beta\mu\alpha} F_{\mu\alpha} = 0 \tag{23}$$

Define dual field $F^*_\mu = \frac{1}{2} \epsilon_{\mu\nu\rho} F^{\nu\rho}$

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Now use $\epsilon^{\nu}_{\mu\beta}\epsilon^{\alpha\beta\rho}\partial_\alpha\partial^\mu F^*_\rho = \partial^\mu\partial_\mu F^{*\nu}$ to yield

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- So mass of the excitation is κe^2.Note the dimension in 2+1.
- The theory remains gauge invariant.

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- The physical mass can be identified at nonzero poles at κe.
- The unphysical massless mode decouples with proper gauge choice i.e $\xi \to 0$.

Coupling with proca

$$\mathcal{L}_{TM} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m^2 A^{\mu}A_{\mu} + \frac{1}{2}\mu\epsilon^{\mu\nu\rho}A_{\mu}\partial_{\nu}A_{\rho} \tag{28}$$

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where

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- The gauge field A_μ has got two distinct masses $m_\pm$.

Polarisation-Degree of Freedom

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- Gauge field A_μ has one independent polarization each on $k^2 = m^2_\pm$.

Propagator

$$G_{\mu\nu} = \frac{ig_{\mu\nu}\left(p^2 - m^2\right) + \mu\epsilon_{\mu\nu\alpha}k^{\alpha}}{\left(p^2 - m_+^2\right)\left(p^2 - m_-^2\right)} + \frac{\imath p_{\mu\nu}\left[\mu^2 - (\xi - 1)\left(p^2 - m^2\right)\right]}{\left(p^2 - \xi m^2\right)\left(p^2 - m_+^2\right)\left(p^2 - m_-^2\right)} \tag{43}$$

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- The physical poles at $m_{\pm}$ are identified with two physical massive mode of excitation.
- The unphysical pole at ξm is gauge dependent and can be made zero with $\xi \to 0$.

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The self dual factorisation would be

$$\left(\delta^\mu_\sigma \mp \frac{1}{m} \epsilon^{\lambda\mu}_\sigma \partial_\lambda\right) \left(\delta^\rho_\mu \pm \frac{1}{m} \epsilon^{\nu\rho}_\mu \partial_\nu\right) A_\rho = 0 \tag{45}$$

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Clearly the equation is satisfied by

$$A_\mu = \pm \frac{1}{m} \epsilon_{\mu\nu\rho} \partial^\nu A^\rho \tag{46}$$

Self Duality of Proca

$$\left(\partial_\mu \partial^\mu + m^2\right) A_\sigma = 0 \tag{44}$$

The self dual factorisation would be

$$\left(\delta^\mu_\sigma \mp \frac{1}{m}\epsilon^{\lambda\mu}_{\sigma}\partial_\lambda\right)\left(\delta^\rho_\mu \pm \frac{1}{m}\epsilon^{\nu\rho}_{\mu}\partial_\nu\right) A_\rho = 0 \tag{45}$$

Clearly the equation is satisfied by

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Self Duality of Proca coupled with CS

- Corresponding to each mass there is a self dual field.

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Now

$$\partial_\mu F^{\mu\nu} + m^2 A^\nu + \frac{1}{2} \mu \epsilon^{\nu\alpha\beta} F_{\alpha\beta} = 0 \tag{48}$$

can be factored into

$$\left[\delta^\mu_\sigma \mp \frac{1}{m_\pm} \epsilon^{\lambda\mu}_\sigma \partial_\mu \right] \left[\delta^\rho_\mu \pm \frac{1}{m_\mp} \epsilon^{\nu\rho}_\mu \partial_\nu \right] A_\rho = 0 \tag{49}$$

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Thank You