ALTERNATIVE OPERATOR SELECTION IN QUANTUM MECHANICS

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$$X|\psi\rangle = x|\psi\rangle \tag{1}$$

$$P|\psi\rangle = -\imath\hbar\frac{d}{dx}|\psi\rangle \tag{2}$$

- Now the question is can we select these operators in any other fashion so that the physics will remain invariant??

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$$X|\psi\rangle = x|\psi\rangle \tag{3}$$

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CHANGE OF BASIS

Let $|\tilde{x}\rangle = \exp(\frac{\imath g(X)}{\hbar})|x\rangle = \exp(\frac{\imath g(x)}{\hbar})|x\rangle$ where $f(x)$ is derivative of $g(x)$

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- As the change is unitary it will not affect the inner product.
- All the physics is contained in inner product.
  So physics will remain invariant under this change of basis.

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- So $\langle \tilde{x}|P|\tilde{x}'\rangle = [-\imath\hbar \frac{d}{dx} + f(x)]\delta(x - x')$
- We have an unitary basis change that indeed permits an alternative operator selection!!!!

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- Lets look at the commutator of X and P in its new form. $[X, P] = \imath\hbar$ Its the same relation what we would have got without any change.
- So we can conclude we can select operator anyway unless we violate the fundamental commutation relation. Coz Nature is depicted through this relation.

ACKNOWLEDGEMENT

- The author duly acknowledges his friend Mr.Soubhik Kumar for his partial thought over the problem.

- Thank you all for listenning!!!