ϕ^4 Theory in Large N Limit
Computation of ϕ-ϕ Scattering
and
Finite Temperature Partition Function

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Sridip Pal (09MS002) †[1]
Supervisor: Prof. S. Trivedi and Prof. S. Minwalla, DTP, TIFR

† *Department of Physical Sciences, Indian Institute of Science Education and Research, Mohanpur-741252, Nadia, West Bengal, India*

Abstract

The pivotal point of this project is to study certain class of Quantum Field theories in large N limit where the theory admits SU(N) or O(N) symmetry.The report reviews the elementary ϕ^4 theory in large N limit and finds out the ϕ-ϕ scattering amplitude and the finite temperature partition function.In the process it illustrates how we can simply things using auxiliary field and how to do Renormalisation in path integral formalism by shifting fields.

1 Introduction

Most of the field theories not being exactly solvable, we solve it in a perturbative fashion. Approximate solutions to weakly coupled theories, like QED, can be obtained through perturbative calculation using the coupling constant as the parameter in which the perturbation is done. But strongly coupled theories, like QCD, does not permit this because the perturbation series may fail to converge even asymptotically when the coupling constant is comparable to 1. An alternate technique, which can be applied when such theories possess an $SO(N)$ or

[1]sridippaliiser@gmail.com

$SU(N)$ symmetry, is perturbative expansion in $1/N$. Here we will wish to review this method which is of great importance because it enables us to expand the set of solvable field theories and also, sometimes, serves as a good approximation in case of actual theories. It has been observed that certain field theories admitting SU(N) or O(N) symmetry becomes simpler as N becomes larger and larger and the solution to the the theory do admit an expansion in power series of $\frac{1}{N}$. A good exposition to this technique can be found in [1].

This project was aimed at gaining familiarity with the large N technique, through various model theories and interesting problems. The collaborative work included:

1. Calculation of scattering amplitude of $O(N)$ ϕ^4 theory in the large N limit.
2. Calculation of partition function of $O(N)$ ϕ^4 theory in the large N limit.
3. Calculation of partition function of abelian Chern-Simons-Dirac theory in the large flavour limit.

We will specifically deal with O(N) model with following lagrangian:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a - \frac{1}{2}\mu_0^2\phi^a\phi^a - \frac{1}{8}\frac{g_0}{N}\left(\phi^a\phi^a\right)^2 \tag{1}$$

Now the standard perturbation theory is done in terms of coupling constant g_0 keeping N fixed. But here we will study in different fashion-we will keep g_0 fixed and study the theory in large N limit i.t in terms of series in $\frac{1}{N}$.The large N scheme typically depicts profound and deeper structures of the theory than the corresponding what we do in in ordinary perturbation theory. This is because the leading $\frac{1}{N}$ approximation preserves much more of the non-linearity of the exact theory than does ordinary lowest-order perturbation theory; for example, two-particle unitarity for the four-point function is exact in order $\frac{1}{N}$.[2].The other important point is the ordinary perturbation fails in case of strongly coupled field theories like QCD.The power series does not make any sense. So studying the theory in large N limit is the only sensible way we can approach to find out the solution to the theory.

2 Large N Technique

2.1 What is Large N limit

We will consider a ϕ^4 field theory invariant under O(N) symmetry group and we will study the theory in large N limit i.e we wish to examine the solution to the theory as a perturbative expansion in powers of $\frac{1}{N}$.

The dynamical variables of the theory is a set of N scalar fields ϕ^a with dynamics defined by following lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a - \frac{1}{2}\mu_0^2\phi^a\phi^a - \frac{1}{8}\lambda_0\left(\phi^a\phi^a\right)^2 \tag{2}$$

The Feynman diagrams[1] for the scattering of two mesons of type a (described by ϕ^a) into two mesons of type b in ordinary perturbation theory are as follows:

Fig. 1

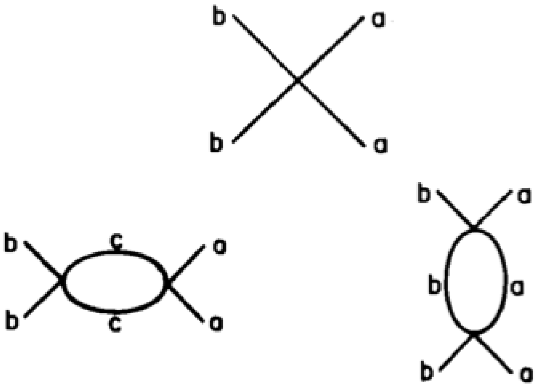

Figure 1: *Feynman diagrams in Ordinary Perturbation*

The first diagram is the born term proportional to λ_0, the second diagram is $O(\lambda_0^2 N)$ because there are N choices for internal index c. The third digram is of order λ_0^2. Now we see if we keep λ_0 fixed then the large N limit nonsensical the second diagram gives us infinity. To make the limit making sense we define $\lambda_0 = \frac{g_0}{N}$ and keep g_0 fixed while taking N to infinity. Hence we have now the first diagram in $O(\frac{g_0}{N})$, the second diagram is of $O(\frac{g_0^2}{N})$ and the third diagram is of $O(\frac{g_0^2}{N^2})$, negligible in compared to first two diagrams in large N limit. Hence in large N technique it is of utmost importance which term is kept fixed and which is not and there lies an important difference between standard perturbation technique and that in powers of $\frac{1}{N}$.

The next figure [1] shows two types of diagram of the order of $\frac{1}{N}$ with various powers of g_0:

Fig. 2

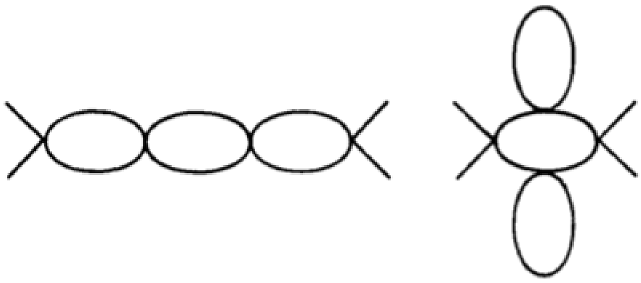

Figure 2: *Order* $\frac{1}{N}$ *graph*

Now if we want a series in powers of $\frac{1}{N}$, it would be really difficult to follow which diagrams contribute in leading order. We can make the power counting simpler by introducing an auxiliary field σ which modifies the lagrangian 2 in following fashion:

$$\mathcal{L}_m = \mathcal{L} + \frac{1}{2}\frac{N}{g_0}\left(\sigma - \frac{1}{2}\frac{g_0}{N}\phi^a\phi^a\right)^2 \tag{3}$$

Note the added term does not have any effect on the dynamics of the theory.The Euler Lagrange equation for σ field does not involve any time derivative, it is a simply a constraint.

$$\sigma = \frac{1}{2}\frac{g_0}{N}\phi^a\phi^a \tag{4}$$

This fact is apparent in path integral formalism too. The path integral over σ is Gaussian and its only job is to multiply the generating functional by an irrelevant constant which does not alter the physical content of the theory. But the feynman rules for the theory will be modified as the new lagrangian looks like:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a - \frac{1}{2}\mu_0^2\phi^a\phi^a + \frac{1}{2}\frac{N}{g_0}\sigma^2 - \frac{1}{2}\sigma\phi^a\phi^a \tag{5}$$

Note now we have following features:

1. The nontrivial interaction does not contain any factor of N.
2. The σ field propagator contains a factor of $\frac{1}{N}$.
3. Every closed ϕ loop carries the same index which is to be summed over giveing a power of N.

Hence the modified lagrangian facilitates the counting of powers of $\frac{1}{N}$. The feynman diagrams of fig 1. would now be modified into following:

Fig. 3

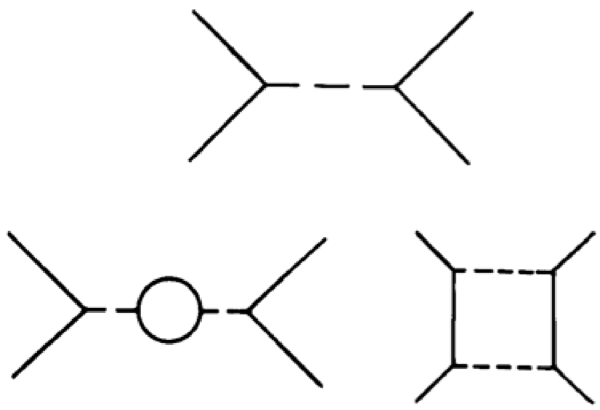

Figure 3: *Feynaman graphs with σ field*

2.2 Effective Action and Calculation of Scattering Amplitude in Large N limit

Now to facilitate the calculation of Scattering Amplitude in large N limit, we will make life more simpler by doing the following trick: we strip the ϕ lines that end on external lines i.e they are not part of a loop. This yields a graph with external σ lines only. Secondly we do all the momentum integrals over closed ϕ loops to generate a graph with σ lines only and the vertex representing the non local interaction between ϕ. As a whole it can be thought of as an effective field theory whose Feynman rules have been derived from an effective action defined by

$$e^{\imath S_{eff}[\sigma]} = \int \prod [d\phi^a] e^{\imath S[\phi^a,\sigma]} \tag{6}$$

In this effective field theory we have

1. The internal and external σ lines carry $\frac{1}{N}$ factor.

2. Each vertex carries a factor of N.

Hence in a graph we have $N^{V-I-E} = N^{1-L-E}$ where V,I,E and L are the no of vertices,internal lines,external lines and loops respectively and we have used the fact that in a feynman diagram $L = I - V + 1$.

The description of effective action in terms of Feynamn diagrams [1] would be following:

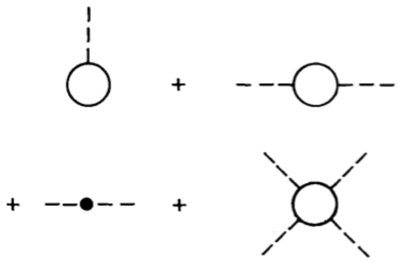

Figure 4: *Feynman Diagrams for* $S_{eff}(\sigma)$

The first graph would give terms proportional to σ while second graph is quadratic which should be added to the third graph, the quadratic term already present in the lagrangian 5.

2.3 ϕ-ϕ Scattering and Renormalisation

Now we will compute the ϕ-ϕ scattering to the leading order i.e O($\frac{1}{N}$). Note each graph has a power of N^{1-L-E}.To compute ϕ-ϕ scattering we need 2 external lines and to leading order we may put $L = 0$ i.e no loop of σ.So the leading order is $N^{1-0-2} = \frac{1}{N}$.

Before delving into calculation we should notice that there is a slight hitch- S_{eff} has an awkward term proportional to σ. We can stick this lollypop like graph anywhere we want and obtain an infinitely different graph of order of $\frac{1}{N}$. To eliminate this infinity we define a shifted field:

$$\sigma' = \sigma - \sigma_0 \tag{7}$$

where σ_0 is the stationary point of S_{eff}. In terms of σ' there is no such linear term.

We can easily write the S_{eff} in terms of shifted field in following manner-just substitute σ in favour of σ' to get

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a - \frac{1}{2}\mu_1^2\phi^a\phi^a + \frac{1}{2}\frac{N}{g_0}\sigma'^2 - \frac{1}{2}\sigma'\phi^a\phi^a + \frac{N}{g_0}\sigma_0\sigma' \tag{8}$$

plus an irrelevent constant where $\mu_1^2 = \mu_0^2 + \sigma_0$. The feynman diagrams for $S_{eff}(\sigma')$ is same as the previous figure except now

1. The internal ϕ lines carry mass μ_1
2. There exists an additional linear vertex due to the last term in 8 which can be used to cancel the linear vertex from the first graph in figure. This fixes σ_0 and eliminate all kind of linear vertex from the calculation. The process is called Renormalisation where the field is redefined to absorb the infinities.

Now we can easily compute ϕ-ϕ scattering to the leading order $\frac{1}{N}$. There are only 3 graphs which can contribute to the process enlisted in following diagrams [1] :

Fig. 5

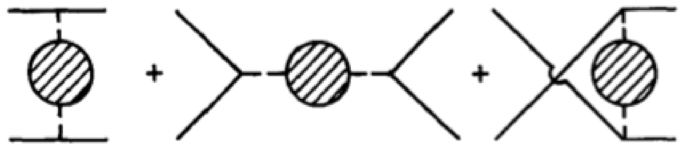

Figure 5: *ϕ-ϕ Scattering*

Note the blobs represent the σ' propagator correct upto the order of $\frac{1}{N}$.So it is not given by $\imath\frac{g_0}{N}$. Hence the problem boils down to calculating the σ' propagator to the order $\frac{1}{N}$.

The following figure enlists the possible graphs which is actually a geometric series:

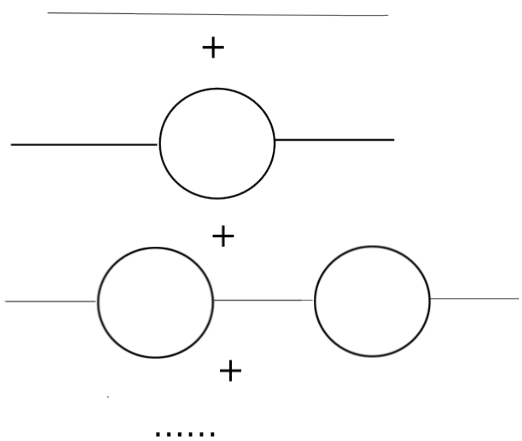

Figure 6: *σ' propagator to the order $\frac{1}{N}$*

Summing up the series we obtain the following result (Note we are doinng this theory in 2+1 dimensions):

$$D^{-1}(p) = N\left[\imath g_0^{-1} + \int \frac{d^3k}{(2\pi)^3}\frac{1}{k^2-\mu_1^2+\imath\epsilon}\frac{1}{(p+k)^2-\mu_1^2+\imath\epsilon}\right] \tag{9}$$

We can write this integral in following manner:

$$I = \imath\int_0^1 dx \int d^3k_E \left[\left(k_E^2+\mu_1^2\right)(1-u) + u\left(p+k_E)^2+\mu_1^2\right)\right]^{-2} \tag{10}$$

where we use the following identity

$$\frac{1}{a_1a_2} = \int_0^1 dx\,(xa_1+(1-x)a_2)^{-2} \tag{11}$$

The integral 10 can be solved to yield:

$$I = \frac{\imath 2\pi^2}{p}cot^{-1}\left(\frac{2\mu_1}{p}\right) \tag{12}$$

So, the final result is:

$$D^{-1}(p) = \imath N\left[g_0^{-1} + \frac{2\pi^2}{p}cot^{-1}\left(\frac{2\mu_1}{p}\right)\right] \tag{13}$$

In massless limit the theory yields:

$$D^{-1}(p) = \imath N\left[g_0^{-1} + \frac{\pi^3}{p}\right] \tag{14}$$

3 Calculation of Finite Temperature Partition Function

3.1 Field Theory at Finite Temperature

In this section, we intend to find out the finite temperature partition function of ϕ^4 theory. Specifically we wish to examine the functional integral representation of partition function. Now we will elucidate what we mean when we say finite temperature field theory.

There is a beautiful correspondence between Quantum Field theory and Statistical Mechanics via wick rotation. Consider the object $\langle\phi_f|e^{\imath Ht}|\phi_i\rangle$. and do a wick rotation i.e $t \to -\imath x_3$ and we land up with $\langle\phi_f|e^{-Hx_3}|\phi_i\rangle$, where H is now wick rotated hamiltonian. The second quantity is encountered when we want to compute partition function $Z(T) = Tr\left(e^{-\beta H}\right)$ where β is the interval in x_3 variable.

Hence we can write

$$Z(T) = Tr\left(e^{-\beta H}\right) = \int_{\phi_0}^{\phi_0} d\tilde{\phi}\langle\phi|e^{-\beta H}|\phi\rangle = \int_{\phi_0}^{\phi_0} d\tilde{\phi}\int D\phi e^{-\int d^3x\mathcal{L}_E} \tag{15}$$

where $\mathcal{L}_E$ is the wick rotated lagrangian.In our case, it is given by following expression:

$$\mathcal{L}_E = \frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a + \frac{1}{2}m^2\phi^a\phi^a + g\left(\phi^a\phi^a\right)^2 \tag{16}$$

Note in the expression 16 the initial and final field configuration is same and we have to integrate over all such configuration. Now the trick is to embed the theory in $\mathcal{R}^2 \times S^1$. S^1 takes care of the boundary condition.If we integrate over all field configuration embedded in S^1 we automatically the the initial and final configuration is same and the integration has been done over all such configuration. S^1 is a neat trick to do away with the restriction imposed by boundary condition.So we have our expression in hand

$$Z(T) = \int D\phi e^{-\int d^3x\mathcal{L}_E} \tag{17}$$

where it is to be understood that the path integral is done over the fields living in the manifold $\mathcal{R}^2 \times S^1$.

We will work in 2+1 dimensions, choosing the physical mass of our theory to be zero at zero temperature. We will perform our calculations consistently in the conformal limit ($gN = \lambda \to \infty$).

3.2 Solution in Large N Limit

The above path integral not being quadratic possesses a problem when we want to solve it analytically. We introduce an auxiliary field σ to make the evaluation simpler in following manner:

$$\mathcal{L}_E \to \mathcal{L}_E - \frac{1}{g}\left(\frac{\sigma}{4} - g\phi^a\phi^a - \frac{m^2}{4}\right)^2$$

Note that addition of this new term will not change the dynamics of the system in any way since the Euler-Lagrange equation of the field yields

$$\frac{\sigma}{4} - g\phi^a\phi^a - \frac{m^2}{4} = 0$$

Having no conjugate momentum, the Euler Lagrange equation corresponding to σ will yield nothing but a constraint equation which equates the additional term to zero.

Our Lagrangian density now becomes (after dropping an irrelevant constant term)

$$\mathcal{L}_E = \frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a + \frac{1}{2}\sigma\phi^a\phi^a + \frac{1}{g}\frac{m^2}{8}\sigma - \frac{1}{g}\frac{\sigma^2}{16}$$

$$Z(T) = \int \mathcal{D}\sigma \mathcal{D}\phi \; e^{-\int d^3x \, (\frac{1}{2}\partial_\mu \phi^a \partial^\mu \phi^a + \frac{1}{2}\sigma\phi^a\phi^a + \frac{N}{\lambda}\frac{m^2}{8}\sigma - \frac{N}{\lambda}\frac{\sigma^2}{16})}$$

We don't have tools powerful enough to perform this integral in general. Hence we make a, hopefully reasonable, assumption. Since the system has rotational and translational symmetry, we assume the scalar field σ to also possess the same, making it a constant. We will solve for it self-consistently. We will also see shortly that at large N, maximum contribution anyway comes from the classical solution.

We notice that the integral is quadratic in ϕ. We may perform the $\mathcal{D}\phi$ integral using the following results :

$$det(\hat{A}) = e^{Tr(ln(\hat{A}))}$$

$$\int \mathcal{D}\phi \; e^{-\frac{1}{2}\int d^4x \, d^4x' \; \phi(x')\hat{A}(x',x)\phi(x)} = det(\hat{A})^{-\frac{1}{2}}$$

$$\begin{aligned} Z(T) &= \int \mathcal{D}\sigma \; det(\partial_\mu\partial^\mu + \sigma)^{-\frac{N}{2}} e^{-\int_V d^3x \, (\frac{N}{\lambda}\frac{m^2}{8}\sigma - \frac{N}{\lambda}\frac{\sigma^2}{16})} \\ &= \int \mathcal{D}\sigma \; e^{-\frac{N}{2}Tr(ln(\partial_\mu\partial^\mu + \sigma))} e^{-\int_V d^3x \, (\frac{N}{\lambda}\frac{m^2}{8}\sigma - \frac{N}{\lambda}\frac{\sigma^2}{16})} \\ &= \int \mathcal{D}\sigma \; e^{-(NV\beta)\left[\left\{\int \frac{d^2p}{(2\pi)^2}\sum_{n=-\infty}^{\infty}\frac{1}{\beta}ln\left(\left(\frac{2\pi}{\beta}n\right)^2 + |\vec{p}|^2 + \sigma\right]\right\} - \frac{1}{\lambda}\frac{m^2}{8}\sigma + \frac{1}{\lambda}\frac{\sigma^2}{16}\right]} \end{aligned} \tag{18}$$

The sum in the last expression is over the quantised momenta lying on S^1.

Inspecting the expression for the partition function, we see that there is an ultraviolet divergence. We will employ a cutoff to rid ourselves of it. Doing so at $T = 0$ will be sufficient, since no new divergences will arise as the temperature is increased (not proved here). Recall that we have prescribed the physical mass of our theory to be zero at $T = 0$, though this is just a matter of choice. Since we will be working in the conformal limit $\lambda \to \infty$, we drop the last term in the exponent.

Notice that as $N \to \infty$, the $\mathcal{D}\sigma$ integral in Equation (18) will have its contribution entirely from the integrand where the term in the bracket is minimum (others are exponentially suppressed). Hence we proceed to find that term's minima.

$$\int \frac{d^2p}{(2\pi)^2}\sum_n \frac{1}{\beta}\frac{1}{\left(\frac{2\pi}{\beta}n\right)^2 + |\vec{p}|^2 + \sigma(T)} = \frac{1}{\lambda}\frac{m^2}{8}$$

As mentioned before, the physical mass of the theory, which is σ, will be set to zero at zero temperature. At finite temperature, the sum does not diverge, so we impose an ultraviolet cutoff only on the integral over the spatial momentum. Taking $\beta \to \infty$ in the above expression,

$$\frac{1}{(2\pi)^3}\int^\Lambda d^2p \int_{-\infty}^{\infty} dp_3 \frac{1}{p_3^2 + |\vec{p}|^2} = \frac{1}{\lambda}\frac{m^2}{8}$$

The integral on the left hand side is $\frac{\Lambda}{4\pi}$ which we will use later.

To compute the partition function at finite temperature, we also need to know the value of σ (it won't be zero at all temperatures). We may solve for it self consistently at the saddle point.

$$\int^{\Lambda} \frac{d^2p}{(2\pi)^2} \sum_{n=-\infty}^{\infty} \frac{1}{\beta} \frac{1}{\left(\frac{2\pi}{\beta}n\right)^2 + |\vec{p}|^2 + \sigma} = \frac{1}{\lambda}\frac{m^2}{8} = \frac{1}{(2\pi)^3} \int^{\Lambda} d^2p \int_{-\infty}^{\infty} dp_3 \frac{1}{p_3^2 + |\vec{p}|^2} \tag{19}$$

$$\int^{\Lambda} \frac{d^2p}{(2\pi)^2} \sum_{n=-\infty}^{\infty} \frac{1}{\beta} \frac{1}{\left(\frac{2\pi}{\beta}n\right)^2 + |\vec{p}|^2 + \sigma} = \frac{1}{(2\pi)^3} \int^{\Lambda} d^2p \int_{-\infty}^{\infty} dp_3 \frac{1}{p_3^2 + |\vec{p}|^2}$$

$$ln\left(\frac{sinh\frac{\beta}{2}\sqrt{\Lambda^2+\sigma}}{sinh\frac{\beta}{2}\sqrt{\sigma}}\right) = \frac{\Lambda\beta}{2} \tag{20}$$

Solving this will yield σ as a function of the cutoff Λ.Specifically in large Λ limit, we have

$$\sigma = \frac{4}{\beta^2}\left(sinh\frac{1}{2}\right)^2 \tag{21}$$

We will now turn to compute the free energy, since its interpretation is more intuitive. Recall that the free energy goes as $F = -\frac{1}{\beta}ln(Z)$. Also, $F = E - TS$, where the zero of E is not fixed. Hence, we will calculate $F = (E - E_0) - TS$. For this, we subtract the free energy at zero temperature (not strictly) from the free energy at temperature T. We will keep only the maximally contributing term mentioned before, so we do away with the $\mathcal{D}\sigma$ integral.

$$ln\left(\frac{Z(T)}{Z_0}\right) = -(NV\beta)\left[\int^{\Lambda} \frac{d^2p}{(2\pi)^2} \sum \frac{1}{\beta} ln\left(\left(\frac{2\pi}{\beta}n\right)^2 + |\vec{p}|^2 + \sigma\right) - \frac{1}{\lambda}\frac{m^2}{8}\sigma\right] + (NV\beta)\int^{\Lambda} \frac{d^3p}{(2\pi)^3} ln\left(p^2\right)$$

We add and subtract the term $(NV\beta)\int \frac{d^3p}{(2\pi)^3} ln\left(p^2+\sigma\right)$ and substitute (19).

$$-\frac{1}{NV\beta} ln\left(\frac{Z(T)}{Z_0}\right) = \int^{\Lambda} \frac{d^2p}{(2\pi)^2} \underbrace{\left\{\sum \frac{1}{\beta} ln\left(\left(\frac{2\pi}{\beta}n\right)^2 + |\vec{p}|^2 + \sigma\right) - \int \frac{dp_3}{2\pi} ln\left(p^2+\sigma\right)\right\}}_{1} + \underbrace{\int^{\Lambda} \frac{d^3p}{(2\pi)^3}\left\{ln\left(\frac{p^2+\sigma}{p^2}\right) - \frac{\sigma}{p^2}\right\}}_{2}$$

The term labelled 2 converges, so we will turn our attention to the term labelled 1. The first of the two terms in term 1 is the finite temperature path integral computation (in 0 dimiension) of the free energy of a harmonic oscillator of $\omega^2 = |\vec{p}|^2 + \sigma$ at temperature $T = \frac{1}{\beta}$, and the second term is the free energy of the same harmonic oscillator at temperature $T = 0$. Now, from simple quantum mechanics, we know this quantity is $\frac{1}{\beta} ln\left(1 - e^{\beta\sqrt{|\vec{p}|^2+\sigma}}\right)$. Hence,

$$
-\frac{1}{NV\beta} ln\left(\frac{Z(T)}{Z_0}\right) = \int^{\Lambda} \frac{d^2p}{(2\pi)^2} \frac{ln\left(1 - e^{\beta\sqrt{|\vec{p}|^2+\sigma}}\right)}{\beta} + \int^{\Lambda} \frac{d^3p}{(2\pi)^3} \left\{ ln\left(\frac{p^2+\sigma}{p^2}\right) - \frac{\sigma}{p^2} \right\}
$$

It is clear that the integral converges in the $\Lambda \to \infty$ limit. The integral may be performed, self-consistently substituting for σ from (21) in large Λ limit.

Note that there are no dimensionless parameters in this theory. We may replace $|\vec{p}|$ by a new dimensionless variable $|\vec{p}|\beta$ and perform the integral. The integral will now become a dimensionless, temperature independent number. It is easy to conclude from this that Free Energy $\propto T^3$.

4 Conclusion

We conclude the report by some remark on how large N should be to employ the scheme without any fail. In real world QCD admits SU(3) symmetry and N is 3. This apparently seems very dull as 3 is not a very large number. But as said by Witten even if N is 3 the parameter in which the series has been expanded i.e $\frac{1}{N} = 0.33$ which can be compared to the coupling constant of QED, as $\frac{e^2}{4\pi} = \frac{1}{137} = 0.3 \Leftrightarrow e = 0.3$. Of course this not imply that QCD is good to study at large N limit but it does show that we should not a priori reject the idea of doing perturbation in $\frac{1}{N}$ series.

The second concluding remark is on what we can do ahead of calculating this scattering amplitude, we could as well evaluate this integral which is convergent, study the scattering amplitude it defines. We could take μ_1^2 to be negative to study the spontaneous symmetry breaking, could also worry about the higher order correction to the amplitude.We can also go on and check the unitarity of S matrix to the leading order in $\frac{1}{N}$.

The path integral representation of the partition function was successfully derived and applied in case of the $O(N)$ version of the ϕ^4 theory. The cubic dependence of the free energy on temperature, an expected feature of a theory with no dimensionless parameters in the conformal limit, was observed, lending confidence to the the consistency of the solution.

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